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NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

WEIGHT OPTIMUM ARCH STRUCTURES

by

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December 1990

Thesis Advisor D. Salinas

REPORT DOCUMENTATION PAGE

Form Approved OMB No 0704-0188

1a REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b RESTRICTIVE MARKINGS
2a SECURITY CLASSIFICATION AUTHORITY	3 DISTRIBUTION / AVAILABILITY OF REPORT: Approved for public release; distribution unlimited
2b DECLASSIFICATION / DOWNGRADING SCHEDULE	
4 PERFORMING ORGANIZATION REPORT NUMBER(S)	5 MONITORING ORGANIZATION REPORT NUMBER(S)
6a NAME OF PERFORMING ORGANIZATION: Naval Postgraduate Schl; 6b OFFICE SYMBOL (If applicable): 69	7a NAME OF MONITORING ORGANIZATION: Naval Postgraduate School
6c ADDRESS (City, State, and ZIP Code): Monterey, California 93943-5000	7b ADDRESS (City, State, and ZIP Code): Monterey, California 93943-5000
8a NAME OF FUNDING / SPONSORING ORGANIZATION; 8b OFFICE SYMBOL (If applicable)	9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c ADDRESS (City, State, and ZIP Code)	10 SOURCE OF FUNDING NUMBERS: PROGRAM ELEMENT NO; PROJECT NO; TASK NO; WORK UNIT ACCESSION NO

11 TITLE (Include Security Classification)

Weight Optimum Arch Structures

12 PERSONAL AUTHOR(S)

McDavid, Charles S. LT/USN

13a TYPE OF REPORT	13b TIME COVERED	14 DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Master's Thesis	FROM ______ TO ______	1990 December	121

16 SUPPLEMENTARY NOTATION

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense

17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	Finite Element Method, Arches, Weight Optimization

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION / AVAILABILITY OF ABSTRACT: ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21 ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED	
22a NAME OF RESPONSIBLE INDIVIDUAL: Professor D. Salinas	22b TELEPHONE (Include Area Code): (408) 646-3426	22c OFFICE SYMBOL: 69Sa

(16) Continued: or the U.S. Government.

Approved for public release; distribution is unlimited

Weight Optimum Arch Structures

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING
from the

NAVAL POSTGRADUATE SCHOOL
December 1990

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ABSTRACT

This investigation is concerned with minimum weight designs of arch structures. Using the finite element method, the arch is modeled by contiguous bar-beam elements. Element stiffness coefficients in terms of local degrees of freedom are related to system degrees of freedom through local to global coordinate transformations. After coordinate transformations, element stiffness coefficients are assembled into FEM stiffness equations for the arch structure. An objective function for weight minimization, with constraints on failure, arch geometry, and section dimensions, is minimized by the DOT optimization code. Results are presented for a number of cases.

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A	the cross-sectional area
b_i	width of the i^{th} element
c	the distance from the center line to the outermost fiber of the element; c=h/2
D	the Domain of the problem
DOT	*Design Optimization Tool* software from VMA Engineering
E	Young's Modulus of the arch material
$\underline{f}^{i}$	the bar-beam force vector in the global coordinate system
$\underline{f}^{i'}$	the bar-beam force vector in the elemental coordinate system
$\underline{f}^{ai}$	the bar elemental force vector
$\underline{f}^{bi}$	the beam elemental force vector
F	Concentrated axial force
$\underline{F}^{a}$	the bar system force vector
$\underline{F}^{b}$	the beam system force vector
$\underline{F}^{A}$	the bar system force vector including the boundary term vector $\underline{U}$
$\underline{F}^{B}$	the beam system force vector including the boundary term vectors $\underline{M}$ and $\underline{V}$
FEM	Finite Element Method
$\underline{G}$	the column vector of linear shape functions
h_i	height of the i^{th} element
I	cross-sectional moment of inertia
$\underline{k}^{i}$	the bar-beam elemental stiffness matrix in x-y coordinates
$\underline{k}^{i'}$	the bar-beam elemental stiffness matrix in local coordinates

$\underset{\sim}{k}^{ai}$	the bar elemental stiffness matrix in local coordinates
$\underset{\sim}{k}^{bi}$	the beam elemental stiffness matrix in local coordinates
$\underset{\sim}{K}$	the bar-beam system (global) stiffness matrix
$\underset{\sim}{K}^A$	the bar system (global) stiffness matrix
$\underset{\sim}{K}^B$	the beam system (global) stiffness matrix
l_i	length of the i^{th} element
L	the total length of the given structure
$\pounds$	the differential operator
M	Moment
M_{max}	Maximum Moment
M_o	Concentrated Moment
$\underset{\sim}{M}$	the moment boundary term vector
NEL	the total number of elements
p_x	axial loading
p_y	lateral loading
P	concentrated load
$\underset{\sim}{Q}$	the column vector of cubic shape functions
r	the ratio of the maximum shear stress to the normal stress due to bending; $r = \tau_{max}/\sigma_n$
R	the radius of the arch
R	the Residual function
s	the center-line coordinate of the arch
S_y	yield strength of the arch material
u	axial displacement
$\hat{u}$	the approximate axial displacement

$\underset{\sim}{u}$	the vector of axial displacements
$\underset{\sim}{U}$	the bar equation boundary term vector
v	lateral "displacement"
$\tilde{v}$	the approximate lateral "displacement"
$\underset{\sim}{v}$	the vector of lateral displacements and slopes
V	the shear force
$\underset{\sim}{V}$	the shear force boundary term vector
x	the horizontal axis
y	the vertical axis
$\underset{\sim}{0}$	the zero vector
α_i	the angle the i^{th} element makes with the x-axis
β_i	the perpendicular compliment of α_i
$\underset{\sim}{\delta}^i$	the bar-beam displacement vector in the x-y coordinates
$\underset{\sim}{\delta}^{i'}$	the (6x1) bar-beam displacement vector associated with $\underset{\approx}{k}^{i'}$
δ_{exact}	the exact analytical solution
$\underset{\approx}{\Gamma}^i$	the (6x6) local transformation matrix
Θ	the subtended arc of the arch
σ_a	the normal stress due to bar (axial) behavior
σ_b	the normal stress due to beam (bending) stress
σ_i	the maximum stress developed in the i^{th} element
σ_n	the total normal stress
τ_{max}	the maximum shear stress

Acknowledgment

My thanks and gratitude to Professor Dong Soo Kim for his time and effort in teaching me the ways of DOT and to Professor Salinas for his open door and infinite patience.

I. INTRODUCTION

The arch has been employed in the fabrication of engineering structures for over five thousand years. Its suitability to compressive load design made it especially favored when masonry, not steel, was the principle building material. (Due to masonry being stronger in compression than in tension.) Its elegant shape, more natural and graceful than the straight lines and perpendiculars of traditional man-made structures, made it fashionable among architects and civil engineers. Its influence can be cited in diverse cultures, among which include the Egyptians, Mesopotamians, Romans, (see Figure 1.1), Byzantines, French, Chinese, and English.

A number of investigations on the optimization of arches have been conducted over the years. In 1976, Farshad [Ref. 1], using the calculus of variations, derived optimality conditions in the form of nonlinear partial differential equations for hinged-hinged arches. An augmented functional, comprised of the total potential energy of the system and the objective function, appended to the system via Lagrange multipliers, when minimized with respect to state variables and with respect to design variables yield the system equilibrium equations, and the optimality conditions respectively. Three objective functions were imposed:

- optimal thrust
- minimum length of arch

– minimum volume

The arch span and the loading are specified. The nonlinear system of optimality equations were presented but not solved.

In 1980, Rozvany et al [Ref. 2] considered the problem of arch optimization using the Prager-Shield criteria. Here, the arch was in fact a funicular frame with beams rigidly interconnected to one another. Only statically determinate systems were investigated. The first "arch" with a specified span consisted of two inclined beams with a concentrated load along the center of symmetry. The second investigation dealt with an "arch" consisting of three beam segments, the center segment being horizontal, and inclined members from the hinged supports. Two concentrated loads were applied at the intersections of the inclined members with the horizontal center member. For the single load "arch" it was found that the optimal "arch" develops either bending only or axial forces only in the entire structure depending on the range of the 4L/d ratio, where L is the span of the structure and d is the constant depth of the cross-section. For 4L/d greater than 8, the optimal structure has a height half of the span, and there is only axial force throughout the structure. For 4L/d less than 8, the optimal structure is a straight horizontal beam (i.e., the height is zero), and only bending throughout the structure. The width of the beam segments for the optimal "arch" varies linearly from the hinged support to the center line.

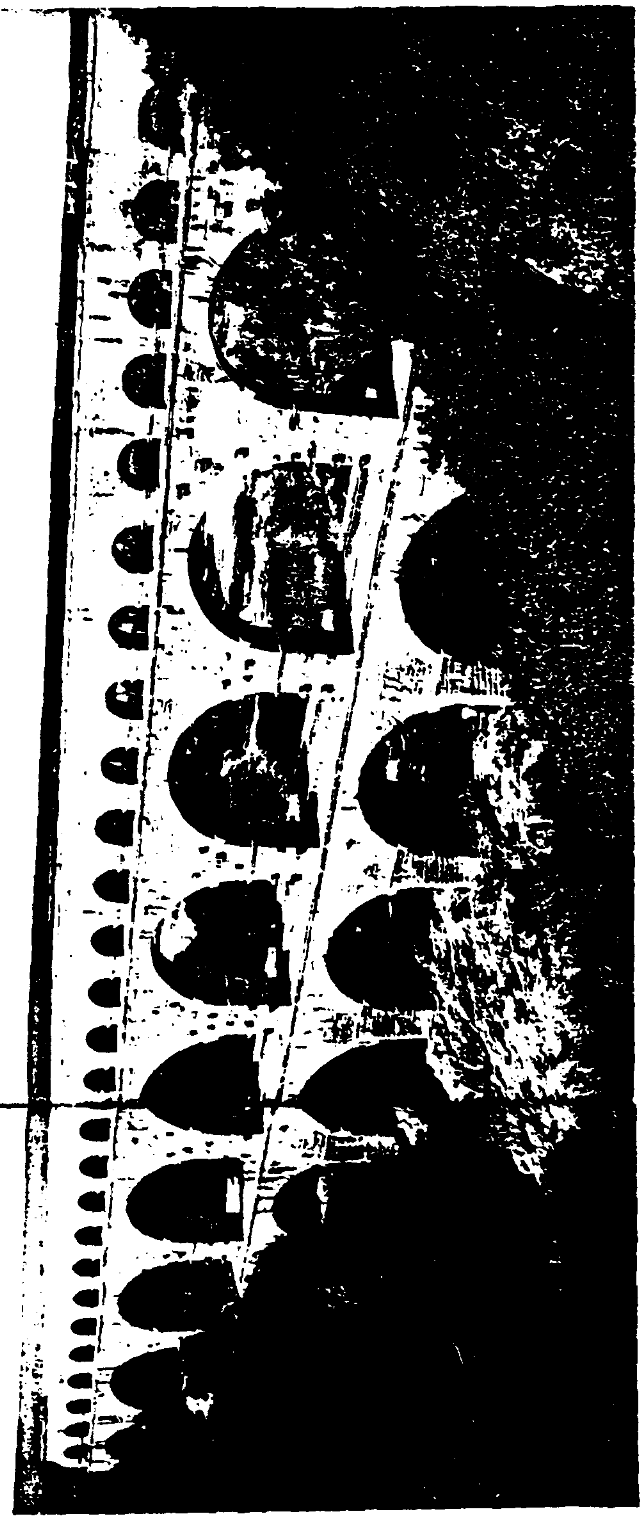


Figure 1.1: Pont du Gard of the Nimes Aqueduct

In a 1980 paper, Lipson et al [Ref. 3] investigated the optimal design of arches using the complex method. Both the arch shape and the cross sectional dimensions are the design variables for the minimum weight structure. Only symmetric arches with constant depth and constant width were considered. The section was taken as a thin walled rectangular tube with vertical and horizontal wall thicknesses as design variables. The arch was approximated with equal length straight beam sections. Thus, each beam segment had three design variables, the two wall thicknesses and the left end vertical location. In addition, the uniform height and uniform width of the rectangular tube were design variables. Side constraints in the form of upper and lower bounds were placed on all of the design variables. A modified version of the complex method of Box was used as the scheme to obtain a "fully-stressed" optimum design. The shape of the arch was taken as a parabola. The optimization algorithm provided the minimum weight parabolic arch for a uniform load over a specified span. It was shown that a parabolic arch with a rise 0.342 times the span length is the optimal parabolic shape for the case of a uniform horizontal load. Deviations within 10% from this rise have negligible effect on the optimal weight. It was further shown that parabolic steel arches will fail due to their own weight at span lengths greater than 1,543 ft. For relatively high arches, the maximum axial thrust, which occurs at the supports, approaches half of the total uniform load. The results of a parametric study of optimal steel arches are presented.

In a 1988 paper, Ang et al [Ref. 4] investigated the optimal shape of an arch under bending and axial compression. The cross-section of the arch was rectangular with specified constant depth and variable width. With the centroidal axis of the arch given by the summation of products of cubic spline functions with linear coefficients, the arch axis is permitted to take on any smooth shape. The linear coefficients in these products are design variables to be determined in the optimization process. The authors considered arches under a uniformly distributed horizontal load with three types of boundaries,

- simply supported-simply supported
- clamped-clamped
- clamped-simply supported

A yield failure constraint was imposed. A new technique for smoothing the objective function is presented. It was shown that the optimal shape of the arch is a parabola with a rise equal to 0.433 time the span of the arch. No other results are presented. It should be noted that the results of [Ref. 3] and [Ref. 4], with regard to the ratio of rise to span for an optimal parabolic arch, do not agree.

In the study of arches, it is necessary to determine how they will be defined. One prevalent school of thought defines an arch as a curved structure, which when supported at both ends and loaded vertically develops horizontal reactions. This is apparently intended to eliminate thick walled curved beams and straight beams which develop (virtually) no horizontal reactions when loaded laterally.

Others define an arch as a curved beam whose cross-sectional dimensions are small relative to its radius of curvature. Hence, the centroidal and neutral axes are assumed to coincide. How the structure is loaded and supported becomes secondary. This description was chosen to facilitate the development of a finite element code capable of generating horizontal and vertical displacements and slopes for an arbitrarily loaded arch. Without the thin depth assumption, complications arise in the calculations of the slopes and displacements because the arch will no longer behave as predicted by the beam equilibrium equation:

$$(EIv'')'' = p_y(s) \tag{1.1}$$

and the bar equilibrium equation:

$$(AEu')' = -p_x(s) \tag{1.2}$$

where E represents Young's Modulus, I the cross-sectional moment of inertia, A the cross-sectional area, v the lateral displacement, u the axial displacement, p_x the axial loading, and p_y the lateral loading.

Once the displacements and slopes are determined, the local stresses can be calculated throughout the member using appropriate stress-displacement relations. Thus able to determine the stress distribution, the arch may be designed to achieve minimum volume (and hence weight) while maintaining the developed stresses below some predefined maximum allowable stress value.

The aim of this study is to "optimize" a linear, elastic, isotropic, and homogeneous arch under a variety of loadings and end conditions. Although these limitations are not physically essential, they were necessary to make the investigation tenable given the time constraints of thesis research activity. Optimization in this investigation will refer to the variance of the cross-sectional geometry to achieve a more uniform stress distribution throughout the member. This results in less material used and a more efficient structure. VMA Engineering's *Design Optimization Tools* [Ref. 5] is used to perform the optimization subject to prescribed constraints on the design variables as well as stress limitations. The objective function to be minimized is the total volume of the arch while maintaining stresses below the yield strength of the arch material. Though a rather simplistic model, it forms a foundation upon which further research into more complex geometries and conditions may be developed.

II. PROBLEM FORMULATION

Perhaps the most common optimization in structural mechanics is the minimization of an element's weight, subject to a specified loading. Such will be the case for this investigation. To make the investigation tenable, the problem needed to be narrowed down in its scope. The assumptions and approximations made in this study are:

- The arch is approximated by a series of straight bar/beam elements which behave according to the beam equation (1.1) and bar equation (1.2). (See Figure 2.1)

- The arch material is isotropic, homogeneous, and linearly elastic.

- The arch's cross-sectional area will always be of a solid rectangular geometry.

- The arch has a constant circular radius of curvature.

- The arch "fails" if its internal stresses exceed the yield strength, S_y.

It should be noted that the third and fourth assumptions are not inherent to the general optimization problem but rather are imposed to limit the scope of this initial investigation. Follow on investigations will relax these restrictions.

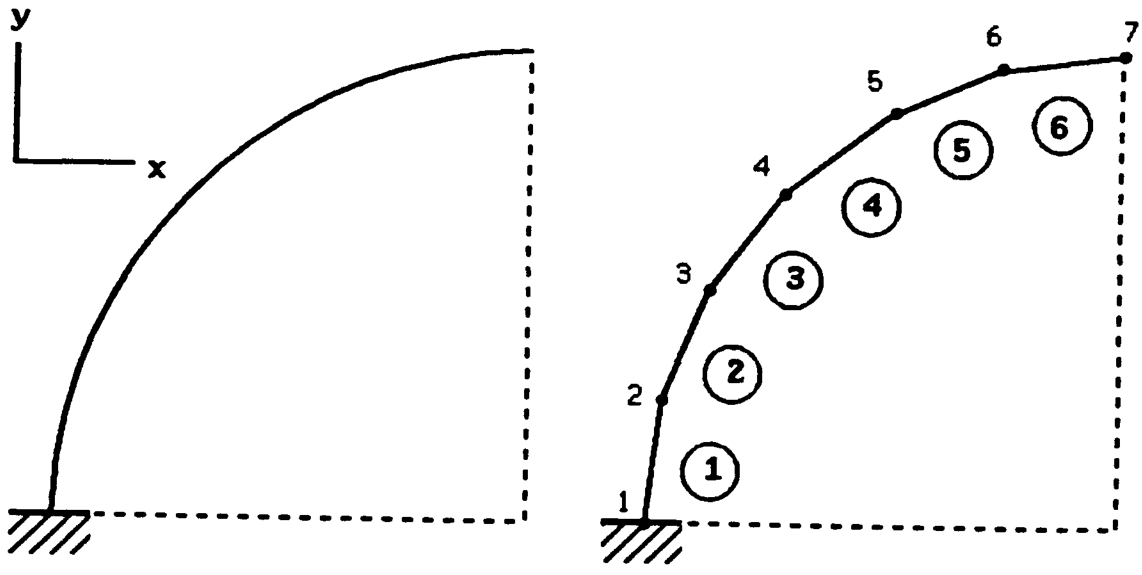


Figure 2.1: Bar-Beam Model of the Arch

Although these suppositions limit the applications for which the optimization can be used, they form a foundation upon which further research can be based.

A. THE OBJECTIVE FUNCTION

The objective of this investigation is the minimization of the structure's weight while maintaining a stress distribution which does not exceed the yield strength. Since the weight of the arch is directly proportional to the volume of material from which it is made, the objective will be satisfied if the total volume of the arch is minimized. That is:

$$\text{Objective} = \text{MIN}\sum_{i=1}^{NEL} b_i h_i l_i \tag{2.1}$$

where b_i, h_i, and l_i represent the width, height, and length of the i^{th} element, respectively, and NEL is the total number of elements. (See Figure 2.1b)

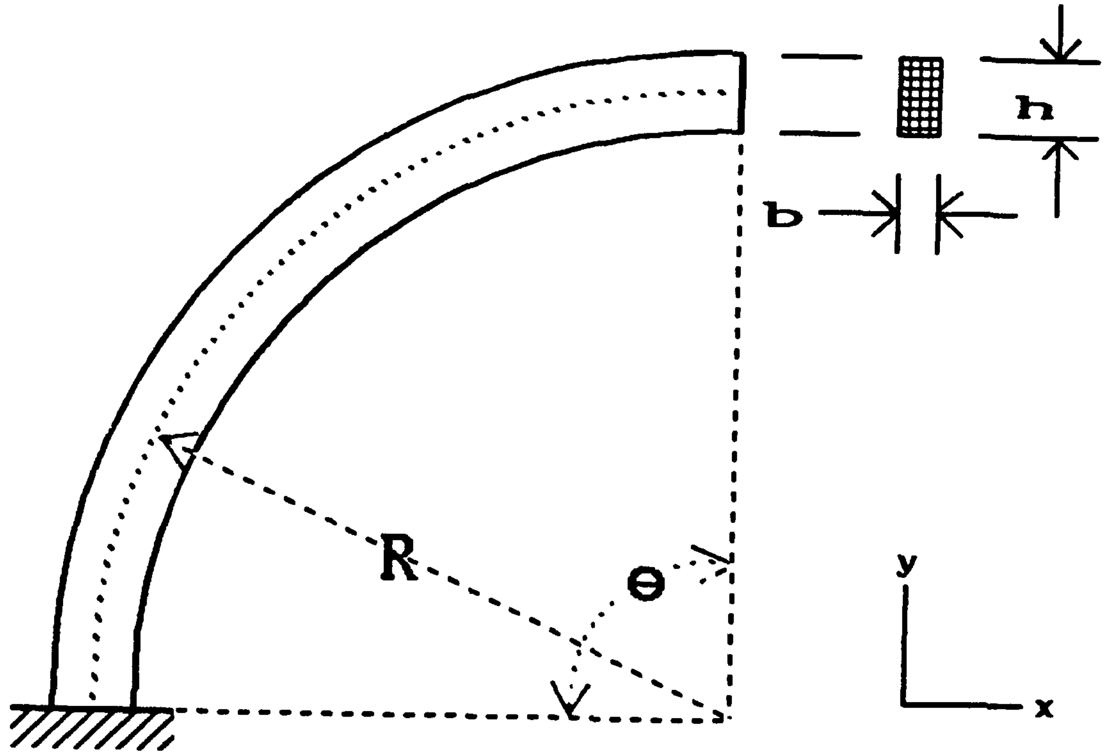


Figure 2.2: Arch Dimensions

With an objective function defined, the next step in the optimization process is to impose any necessary (design) constraints upon the system. The constraints must be provided to represent the assumptions contained in the modeling equations. They should be utilized to avoid undesirable behavior such as buckling and yielding. They may also be used to apply any limitations on behavior as desired by the designer. The constraints employed in this investigation follow.

B. THE STRENGTH CONSTRAINT

This study assumes a linearly elastic arch. Therefore, the applied loading must not cause the structure to exceed the elastic limit of the material from which it is made. Hence, a design constraint which will prevent this mode of "failure by yielding" must be imposed. To fulfill this constraint, the internal stresses developed must remain below the yield strength of the material. Defining the maximum stress developed in the i^{th} element of the arch to be σ_i and the yield strength of the arch material as S_y, this constraint becomes

$$\sigma_i \leq S_y$$

or in dimensionless form:

$$\sigma_i/S_y - 1 \leq 0 \tag{2.2}$$

C. GEOMETRIC CONSTRAINTS

To use the beam and bar equations, limits must be imposed upon the geometric dimensions of the structure. It is compulsory that the depth and width be of at least an order of magnitude smaller than the radius of curvature for the beam and bar equations to be applicable. Hence,

$$h_i \leq R_i/10$$

or in dimensionless form:

$$10h_i/R_i - 1 \leq 0 \tag{2.3}$$

Similarly, as the width increases, the arch approaches the geometry of a shell. To maintain the geometry of an arch, an imposition upon the width dimension is also necessary. To avoid shell behavior, a third constraint is imposed

$$b_i \leq 3h_i$$

or in dimensionless form:

$$b_i/3h_i - 1 \leq 0 \tag{2.4}$$

D. SIDE CONSTRAINTS

Finally, side constraints must be assigned to the dimensions of the design variables. For the sake of simplicity, this investigation will only take up the variation of the cross-sectional width dimension b_i. The side constraints must ensure real solutions are obtained, i.e., the arch is a physical object and therefore h_i and b_i cannot be less than a realistic finite value. The side constraints chosen to reflect these limitations include:

$$0.03 \text{ in.} \leq h_i \qquad (2.5)$$

$$0.03 \text{ in.} \leq b_i \qquad (2.6)$$

Other constraints could have also been considered. Global buckling and local crippling are to name but two. However, the cases to be studied do not warrant such a thorough delineation. Therefore, the design and side constraints have been limited to those cited.

E. OPTIMIZATION SOFTWARE

With a multitude of preprogrammed optimization routines available, the *Design Optimization Tools* (DOT) software was chosen. Its selection was based upon availability, ease of use, and reputation. DOT is a FORTRAN 77 optimization software package available from VMA Engineering. To perform a variety of optimization tasks, DOT uses:

- The Modified Method of Feasible Directions,
- Broydon-Fletcher-Goldfarb-Shanno (BFGS) Variable Metric Method,
- Polynomial Interpolation with bounds, and
- Sequential Linear Programming (SLP)

A user provided "main" program is used to input the variables required by DOT. DOT in turn calls a user provided subroutine which defines the objective function, design constraints, and design variable side constraints. DOT iteratively evaluates the objective function, refining the design variables until the optimal solution is obtained.

The parameters used to calculate the objective function and constraints must be known before any optimization can occur. The variables from equations (2.1) through (2.6) include:

- The number of elements used, NEL.
- The arch radius of curvature, R.
- The height of the i^{th} element, h_i.
- The length of the i^{th} element, l_i.
- The width of the i^{th} element, b_i.
- The yield strength of the material from which the arch is made, S_y.
- The stress at the i^{th} node, σ_i.

Of these terms, the number of elements is the choice of the designer. The arch radius of curvature and height are constant throughout the span of the arch and are defined by the problem. For simplicity, the length of each element is made uniform such that:

$$l_i = \Theta R/(NEL) \tag{2.7}$$

where Θ represents the subtended arc of the arch. The yield strength is determined by the material used to build the arch and the width is the design variable to be determined by DOT.

The stress distribution is not so readily available. However, using the beam and bar equations, a finite element scheme can be developed to determine the arch's displacements and slopes due to a given loading. Knowing how the displacements and slopes change throughout the arch, the stress at a given point can be calculated and the optimization performed.

III. STRESS ANALYSIS

The objective of this optimization is to minimize the total weight (volume) of a load bearing arch subject to specified constraints. The strength constraint requires that the stress at any point does not exceed the yield strength of the arch material. To avoid violating this constraint, the value of the stresses at any point must be known. With this requirement, the stress development is pursued.

The normal stress at any point in the arch is composed of normal stresses due to bending (bending stress) and normal stresses due to the axial forces (axial stress) acting upon the individual elements. Figure (3.1) depicts these force interactions. The total normal stress is the algebraic sum of these components.

$$\sigma_n = \sigma_b + \sigma_a \tag{3.1}$$

The arch can also develop shear stresses due to shear forces. Due to the geometric constraint defined by equation (2.3), the shear stresses turn out to be insignificant when compared to the normal stresses. Consequently, the shear stresses are ignored.[1]

To calculate the normal stress components, we must first determine how the elements behave. For a simple straight beam element, the maximum normal stress due to bending occurs at the outer fibers and is given by

$$\sigma_b = Mc/I$$

1. See Appendix A for a justification of the omission of the shear stresses.

or in terms of the beam equation:

$$\sigma_b = (EIv'')c/I$$

which reduces to:

$$\sigma_b = Ecv'' \tag{3.2}$$

where c is the distance from the neutral axis to the outer fiber of the beam, that is c=h/2. See Figure (3.1b).

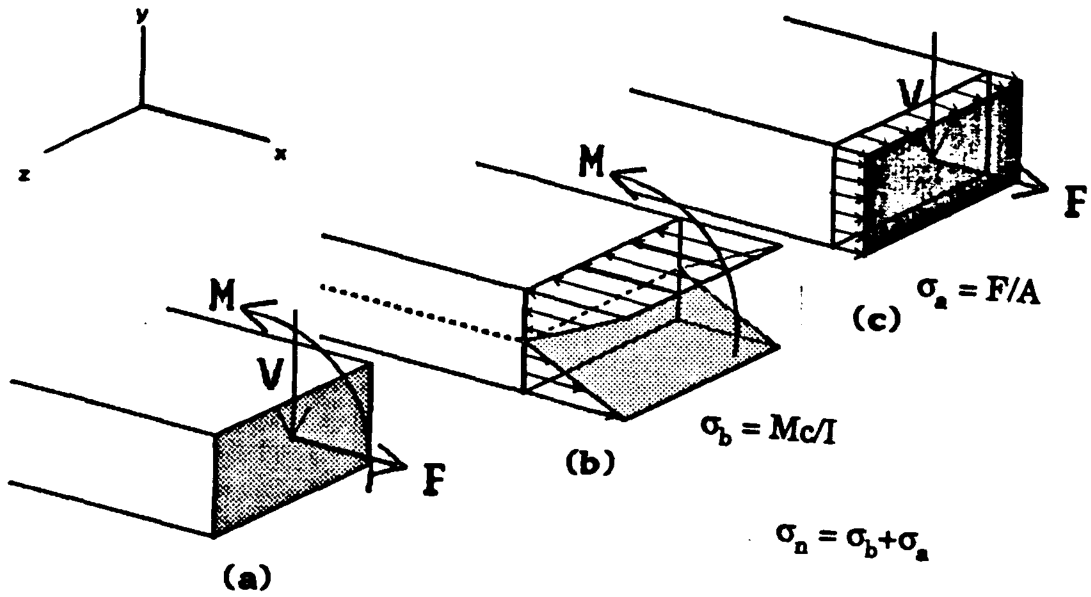


Figure 3.1: Normal Stresses due to Bending and Axial Force

In the same manner, the normal stresses due to axial behavior can be determined. The uniform normal stress due to axial forces, F, acting upon a bar can be defined by:

$$\sigma_a = F/A$$

or in terms of the bar equation,

$$\sigma_a = (AEu')/A$$

which reduces to:

$$\sigma_a = Eu' \tag{3.3}$$

See Figure (3.1c).

Substituting equations (3.2) and (3.3) into equation (3.1) yields the linear equation

$$\sigma_n = Ecv'' + Eu'$$

or simply

$$\sigma_n = E(cv'' + u') \tag{3.4}$$

where v'' and u' are to be determined from the solution of equations (1.1) and (1.2).

From this development, we see the normal stress is a function of Young's Modulus, the height of the beam, the first derivative of the axial displacement, and the second derivative of the lateral displacement. Using the Galerkin finite element method, approximate values of u' and v'' can be determined. With these values, the stress distribution can be calculated using Equation (3.4) and the optimization may then be conducted.

IV. FINITE ELEMENT ANALYSIS

In order to determine the stresses developed for a given loading, the values of u' and v'' must be determined. These derivatives can be found by solving the beam and bar equations using the Galerkin finite element method (FEM). The Galerkin FEM is capable of directly solving systems of linear differential equations while preserving their symmetry.

A. THE BEAM EQUATION DEVELOPMENT

The beam equation (1.1) is a fourth order linear differential equation requiring C^1 continuity. Therefore, a family of cubic shape functions are necessary to maintain function and slope continuity. With this in mind, the Galerkin FEM is performed on the beam equation. A finite element method is an approximation method which transforms the differential equation of a continuous system into a system of linear algebraic equations. The method begins by a discretization, that is a partition, of the continuous domain into a segmented domain of NEL elements. Thereafter, a three step process takes place.

The first step is to form an approximate solution $\tilde{v}$,

$$v \approx \tilde{v} = Q^T \underset{\sim}{v} \tag{4.1}$$

where v is the exact solution in continuous space of the beam equation, $\tilde{v}$ is the approximate solution in discrete space, Q^T is the transpose of the column vector of cubic shape functions which have the Kronecker delta property, and $\underline{v}$ is the vector of coefficients of lateral displacements and slopes.

The second step is to form the residual of the approximation where:

$$R = \pounds(\tilde{v}) - p \tag{4.2}$$

where £ denotes the system operator, in this case being the beam equation such that

$$\pounds(v) = [EI(v)'']''$$

Substituting the beam equation (1.1) and the equation (4.1) into equation (4.2) yields:

$$R = \left[EI(Q^T\underline{v})''\right]'' - p_y(s) \tag{4.3}$$

The third step is to form the Galerkin Equations,

$$\int Q(R)ds = \underline{0} \tag{4.4}$$

where $\underline{0}$ represents a vector whose values are zero. Substituting equation (4.3) into equation (4.4) yields:

$$\int_D Q\left[EI(Q^T\underline{v})''\right]''ds - \int_D Qp_y(s)\,ds = \underline{0} \tag{4.5}$$

Performing two successive integrations by parts upon equation (4.5) yields:

$$Q\left[EI(Q^T\underline{v})''\right]'\Big|_B - Q'EI(Q^T\underline{v})''\Big|_B + \int_D Q''EI(Q^T\underline{v})''ds - \int_D Qp_y(s)\,ds = \underline{0} \tag{4.6}$$

where B denotes the boundary values of the structure at each end point.

Since the coefficients of the solution vector are constants, equation (4.6) may be rewritten as:

$$Q\left[EI(Q^T)''\right]'\underline{v}\Big|_B - Q'EI(Q^T)''\underline{v}\Big|_B + \int_D Q''EI(Q^T)''ds\,\underline{v} - \int_D Qp_y(s)\,ds = \underline{0} \tag{4.7}$$

Using shear given by $V=EIv'''$ and moment given by $M=EIv''$, we define the boundary term load vectors $\underset{\sim}{V}$ and $\underset{\sim}{M}$ as

$$\underset{\sim}{V} = \underset{\sim}{Q}\left[EI(\underset{\sim}{Q}^T)''\right]'\underset{\sim}{v}\Big|_B \tag{4.8a}$$

and

$$\underset{\sim}{M} = \underset{\sim}{Q}'EI(\underset{\sim}{Q}^T)''\underset{\sim}{v}\Big|_B \tag{4.8b}$$

In addition, we define the system stiffness matrix $\underset{\sim}{K}^B$ as

$$\underset{\sim}{K}^B = \int_D \underset{\sim}{Q}''EI(\underset{\sim}{Q}^T)''ds \tag{4.8c}$$

and the system force vector $\underset{\sim}{F}^b$ as,

$$\underset{\sim}{F}^b = \int_D \underset{\sim}{Q}p_y(s)\,ds \tag{4.8d}$$

and upon substituting equations (4.8) into equation (4.7) we obtain the system of linear algebraic equations:

$$\underset{\sim}{V} - \underset{\sim}{M} + \underset{\sim}{K}^B\underset{\sim}{v} - \underset{\sim}{F}^b = \underset{\sim}{0} \tag{4.9}$$

Moving the applied internal excitation and boundary terms to the right-hand side, such that

$$\underset{\sim}{K}^B\underset{\sim}{v} = \underset{\sim}{F}^b + \underset{\sim}{M} - \underset{\sim}{V} \tag{4.10}$$

and defining the load vector of internal and external applied loads as

$$\underset{\sim}{F}^B = \underset{\sim}{F}^b + \underset{\sim}{M} - \underset{\sim}{V} \tag{4.11}$$

equation (4.10) simplifies to the linear system:

$$\underset{\sim}{K}^B\underset{\sim}{v} = \underset{\sim}{F}^B \tag{4.12}$$

where the system bending stiffness matrix, $\underset{\sim}{K}^B$ is constructed from the union of the i=1,...,NEL elemental bending stiffness matrices, $\underset{\sim}{k}^{bi}$ and the system bending force vector $\underset{\sim}{F}^b$ is formed from the union of the i=1,...,NEL elemental bending force vectors $\underset{\sim}{f}^{bi}$.

B. THE BAR EQUATION DEVELOPMENT

The FEM application to the bar equation is similar to that previously conducted with the beam equation. The bar equation, however, is a second order linear differential equation requiring C^0 continuity. Hence, only a family of linear shape functions are necessary to maintain function continuity in the FEM development.

Again, the basic steps of the Galerkin Method are conducted.

First, the approximate solution $\bar{u}$ is formed:

$$u \approx \bar{u} = \underset{\sim}{G}^T \underset{\sim}{u} \tag{4.13}$$

where u is the exact solution of the bar equation, $\bar{u}$ is the approximate solution, $\underset{\sim}{G}^T$ is the transposed column of linear shape functions with the Kronecker delta property, and $\underset{\sim}{u}$ is the vector of coefficients of axial displacements.

Second, the residual is formed:

$$R = \pounds(\bar{u}) + p \tag{4.14}$$

where £ pertains to the differential operator of the bar equation, that is,

$$\pounds(u) = [AE(u)']'$$

Third, the Galerkin equations are formed:

$$\int_D \underset{\sim}{G}(R)ds = \underset{\sim}{0} \tag{4.15}$$

Substituting equation (4.13) and the bar equation (1.2) into equation (4.14) yields:

$$R = \left[AE(\underset{\sim}{G}^T\underset{\sim}{u})'\right]' + p_x(s) \tag{4.16}$$

Substituting equation (4.16) into equation (4.15) yields:

$$\int_D \underset{\sim}{G}\left[AE(\underset{\sim}{G}^T\underset{\sim}{u})'\right]'ds + \int_D \underset{\sim}{G}p_x(s)\,ds = \underset{\sim}{0} \tag{4.17}$$

Performing a single integration by parts upon equation (4.17) yields:

$$AE\underset{\sim}{G}(\underset{\sim}{G}^T\underset{\sim}{u})'\big|_B - \int_D \underset{\sim}{G}'(AE(\underset{\sim}{G}^T\underset{\sim}{u})')ds + \int_D \underset{\sim}{G}p_x(s) = \underset{\sim}{0} \tag{4.18}$$

Again, removing the solution vector $\underset{\sim}{u}$ of constant coefficients outside of the integral yields:

$$AE\cdot\underset{\sim}{G}(\underset{\sim}{G}^T)'\underset{\sim}{u}\big|_B - \int_D \underset{\sim}{G}'[AE(\underset{\sim}{G}^T)']ds\ \underset{\sim}{u} + \int_D \underset{\sim}{G}p_x(s) = \underset{\sim}{0} \tag{4.19}$$

Recalling that the axial force F is AEu′, we define the boundary term vector $\underset{\sim}{U}$,

$$\underset{\sim}{U} = AE\cdot\underset{\sim}{G}(\underset{\sim}{G}^T)'\underset{\sim}{u}\ \big|_B \tag{4.20A}$$

the system stiffness matrix of the bar $\underset{\sim}{K}^A$

$$\underset{\sim}{K}^A = \int_D \underset{\sim}{G}'(AE(\underset{\sim}{G}^T))ds \tag{4.20b}$$

and the force vector associated with internal loading p_x, as

$$\underset{\sim}{F}^a = \int_D \underset{\sim}{G}p_x(s) \tag{4.20c}$$

We obtain upon substituting equations (4.20) into equation (4.19),

$$\underset{\sim}{U} - \underset{\sim}{K}^A\underset{\sim}{u} + \underset{\sim}{F}^a = \underset{\sim}{0} \tag{4.21}$$

Taking all the internal excitation $\underset{\sim}{F}^a$ terms and boundary load terms $\underset{\sim}{U}$ to the right-hand side of equation (4.21) yields:

$$\underset{\sim}{K}^A\underset{\sim}{u} = \underset{\sim}{F}^a + \underset{\sim}{U} \tag{4.22}$$

Defining the load vector $\underset{\sim}{F}^A$ as

$$\underset{\sim}{F}^A = \underset{\sim}{F}^a + \underset{\sim}{U} \tag{4.23}$$

and substituting equation (4.23) into equation (4.22) yields the linear system:

$$\underset{\sim}{K}^A\underset{\sim}{u} = \underset{\sim}{F}^A \tag{4.24}$$

where

$$\underset{\sim}{K}^A = \cup\underset{\sim}{k}^a \tag{4.25}$$

that is, the system axial stiffness matrix, $\underset{\sim}{K}^A$ is constructed from the union of the elemental axial stiffness matrices, $\underset{\sim}{k}^{ai}$, and the system axial force vector $\underset{\sim}{F}^a$ is formed from the union of the elemental axial force vectors $\underset{\sim}{f}^{ai}$.

C. THE ELEMENTAL STIFFNESS MATRIX

In the FEM code, the global (or system) Galerkin FEM equations (4.12) and (4.24) are actually constructed from element considerations as follows. First, the arch is divided into NEL straight beam-bar elements as illustrated in Figure (2.1). The stress analysis program contains a subroutine which constructs the elemental beam and bar stiffness matrices, that is, the $\underset{\sim}{k}^{bi}$ matrices for bending and $\underset{\sim}{k}^{ai}$ matrices for axial stiffness. The bending and axial elemental force vectors, $\underset{\sim}{f}^{bi}$ and $\underset{\sim}{f}^{ai}$ are also determined within this subroutine. Figure (4.1) illustrates the degrees of freedom in which these elemental forces (and displacements) act.

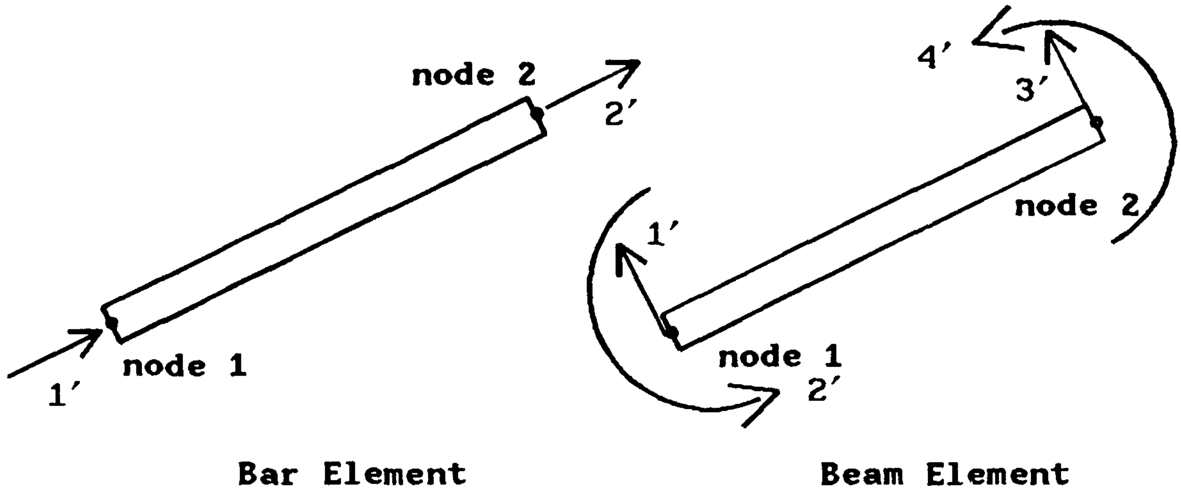


Figure 4.1: Beam and Bar Element Degrees of Freedom

The (4x4) $\underset{\approx}{k}^{bi}$ and (2x2) $\underset{\approx}{k}^{ai}$ matrices of the form:

$$\underset{\approx}{k}^{bi} = \begin{bmatrix} k_{11}^{bi} & k_{12}^{bi} & k_{13}^{bi} & k_{14}^{bi} \\ k_{21}^{bi} & k_{22}^{bi} & k_{23}^{bi} & k_{24}^{bi} \\ k_{31}^{bi} & k_{32}^{bi} & k_{33}^{bi} & k_{34}^{bi} \\ k_{41}^{bi} & k_{42}^{bi} & k_{43}^{bi}\, i & k_{44}^{bi} \end{bmatrix} \qquad \underset{\approx}{k}^{ai} = \begin{bmatrix} k_{11}^{ai} & k_{12}^{ai} \\ k_{21}^{ai} & k_{22}^{ai} \end{bmatrix}$$

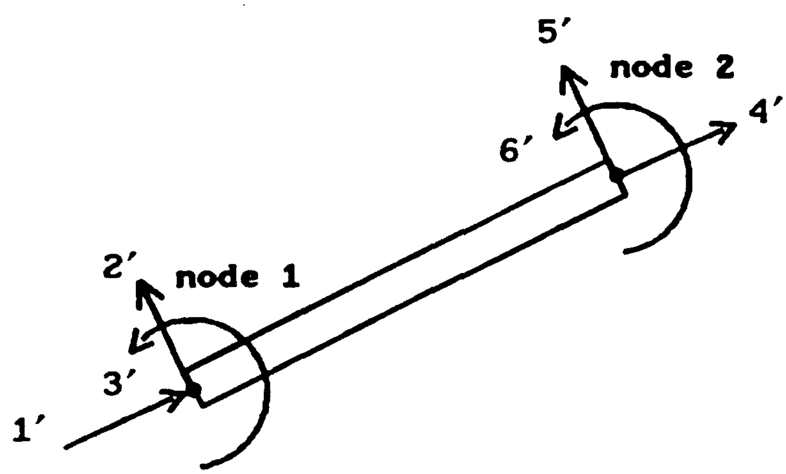
are combined to form a single (6x6) stiffness matrix, $\underset{\approx}{k}^{i}$. This is accomplished by redefining the beam and bar degrees of freedom in the following manner:

– Redefine the bar local degrees of freedom 1′ and 2′, which refer to the axial displacements at each end, as 1′ and 4′ respectively.

- Redefine the beam local degrees of freedom 1′, 2′, 3′, and 4′, which refer to the lateral displacement and beam slope at each end as 2′, 3′, 5′, and 6′ respectively. The redefined degrees of freedom are illustrated by Figure (4.2)

- Place the respective components of the beam matrices $\underset{\approx}{k}^{bi}$ and bar matrices $\underset{\approx}{k}^{ai}$ into the elemental stiffness matrix $\underset{\approx}{k}^{i}$ where:

$$\underset{\approx}{k}^{i} = \begin{bmatrix} k_{11}^{ai} & 0 & 0 & k_{12}^{ai} & 0 & 0 \\ 0 & k_{11}^{bi} & k_{12}^{bi} & 0 & k_{13}^{bi} & k_{14}^{bi} \\ 0 & k_{21}^{bi} & k_{22}^{bi} & 0 & k_{23}^{bi} & k_{24}^{bi} \\ k_{21}^{ai} & 0 & 0 & k_{22}^{ai} & 0 & 0 \\ 0 & k_{31}^{bi} & k_{32}^{bi} & 0 & k_{33}^{bi} & k_{34}^{bi} \\ 0 & k_{41}^{bi} & k_{42}^{bi} & 0 & k_{43}^{bi} & k_{44}^{bi} \end{bmatrix} \qquad (4.26)$$



Beam-bar Element

Figure 4.2: Beam-Bar Element Degrees of Freedom

Defining the elemental displacements and forces in a similar manner, the elemental displacement vector becomes:

$$(\underline{\delta}^{i'})^T = <\delta_1^{i'}, \delta_2^{i'}, \delta_3^{i'}, \delta_4^{i'}, \delta_5^{i'}, \delta_6^{i'}> \tag{4.27}$$

where for the i^{th} element:

$\delta_1^{i'}$ = the axial displacement at node 1

$\delta_2^{i'}$ = the lateral displacement at node 1

$\delta_3^{i'}$ = the beam slope at node 1

$\delta_4^{i'}$ = the axial displacement at node 2

$\delta_5^{i'}$ = the lateral displacement at node 2

$\delta_6^{i'}$ = the beam slope at node 2

and are illustrated in Figure (4.2).

In the same manner, the elemental force vector is redefined as:

$$(\underline{f}^{i'})^T = <f_1{}^{i'}, f_2{}^{i'}, f_3{}^{i'}, f_4{}^{i'}, f_5{}^{i'}, f_6{}^{i'}> \tag{4.28}$$

where for the i^{th} element:

$f_1{}^{i'}$ = the axial force at node 1

$f_2{}^{i'}$ = the lateral force at node 1

$f_3{}^{i'}$ = the moment at node 1

$f_4{}^{i'}$ = the axial force at node 2

$f_5{}^{i'}$ = the lateral force at node 2

$f_6{}^{i'}$ = the moment at node 2

also illustrated in Figure (4.2).

With these developments, the elemental system of equations for the beam-bar element becomes:

$$\begin{bmatrix} AE/l_i & 0 & 0 & -AE/l_i & 0 & 0 \\ 0 & 12EI/l_i^3 & 6EI/l_i^2 & 0 & -12EI/l_i^3 & 6EI/l_i^2 \\ 0 & 6EI/l_i^2 & 4EI/l_i & 0 & -6EI/l_i^2 & 2EI/l_i \\ -AE/l_i & 0 & 0 & AE/l_i & 0 & 0 \\ 0 & -12EI/l_i^3 & -6EI/l_i^2 & 0 & 12EI/l_i^3 & -6EI/l_i^2 \\ 0 & 6EI/l_i^2 & 2EI/l_i & 0 & -6EI/l_i^2 & 4EI/l_i \end{bmatrix} \times \begin{Bmatrix} \delta_1^{i'} \\ \delta_2^{i'} \\ \delta_3^{i'} \\ \delta_4^{i'} \\ \delta_5^{i'} \\ \delta_6^{i'} \end{Bmatrix} = \begin{Bmatrix} f_1^{i'} \\ f_2^{i'} \\ f_3^{i'} \\ f_4^{i'} \\ f_5^{i'} \\ f_6^{i'} \end{Bmatrix} \tag{4.29}$$

or simply

$$\underline{k}^{i'}\underline{\delta}^{i'} = \underline{f}^{i'} \tag{4.30}$$

Prior to incorporation into the global matrix, a coordinate transformation from local to global coordinates is undertaken.

D. COORDINATE TRANSFORMATION

Were all the elements of the same orientation with respect to one another as it is for a straight beam, a global system of equations could be directly constructed. For the arch, however, none of the elements share the same orientation. This necessitates the conversion of all elemental displacements and forces to a system of global displacements and forces. For a reference coordinate system, the horizontal and vertical axes of the arch were chosen. (See Figure 4.3) Defining the angle the i^{th} element makes with the horizontal axis as α_i, and the 90° complement of this angle as β_i, the following coordinate relationship between local and global "displacements" and "forces" exist,

$$
\begin{aligned}
\delta_1^{i'} &= \delta_1^i \cos(\alpha_i) + \delta_2^i \cos(\beta_i) \\
\delta_2^{i'} &= -\delta_1^i \cos(\beta_i) + \delta_2^i \cos(\alpha_i) \\
\delta_3^{i'} &= \delta_3^i \\
\delta_4^{i'} &= \delta_4^i \cos(\alpha_i) + \delta_5^i \cos(\beta_i) \\
\delta_5^{i'} &= -\delta_4^i \cos(\beta_i) + \delta_5^i \cos(\alpha_i) \\
\delta_6^{i'} &= \delta_6^i
\end{aligned}
\tag{4.31}
$$

and

$$
\begin{aligned}
f_1{}^{i'} &= f_1{}^i \cos(\alpha_i) + f_2{}^i \cos(\beta_i) \\
f_2{}^{i'} &= f_1{}^i \cos(\alpha_i) + f_2{}^i \cos(\beta_i) \\
f_3{}^{i'} &= f_3{}^i \\
f_4{}^{i'} &= f_4{}^i \cos(\alpha_i) + f_5{}^i \cos(\beta_i) \\
f_5{}^{i'} &= f_4{}^i \cos(\alpha_i) + f_5{}^i \cos(\beta_i) \\
f_6{}^{i'} &= f_6{}^i
\end{aligned}
\tag{4.32}
$$

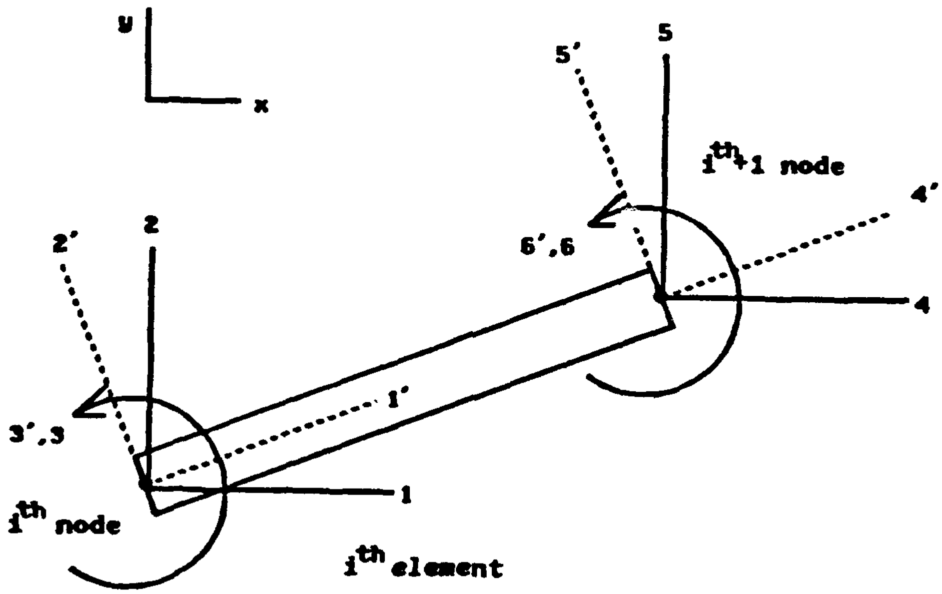


Figure 4.3: Displacement & Force Coordinate Transformations

Defining $\underset{\sim}{\Gamma}^{i}$ as the transformation matrix for the i^{th} element which is capable of performing the appropriate coordinate transformations from local elemental coordinates to global system coordinates, the matrix becomes:

$$\Gamma^{i} = \begin{bmatrix} \cos(\alpha_i) & \cos(\beta_i) & 0 & 0 & 0 & 0 \\ -\cos(\beta_i) & \cos(\alpha_i) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha_i) & \cos(\beta_i) & 0 \\ 0 & 0 & 0 & -\cos(\beta_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.33}$$

and the relation between local and global "displacements" is

$$\begin{Bmatrix} \delta_1^i \\ \delta_2^i \\ \delta_3^i \\ \delta_4^i \\ \delta_5^i \\ \delta_6^i \end{Bmatrix} = \begin{bmatrix} \cos(\alpha_i) & \cos(\beta_i) & 0 & 0 & 0 & 0 \\ -\cos(\beta_i) & \cos(\alpha) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\alpha_i) & \cos(\beta_i) & 0 \\ 0 & 0 & 0 & -\cos(\beta_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \delta_1^{i'} \\ \delta_2^{i'} \\ \delta_3^{i'} \\ \delta_4^{i'} \\ \delta_5^{i'} \\ \delta_6^{i'} \end{Bmatrix} \tag{4.34}$$

A similar relation between local forces $f^{i'}$ and global forces f^{i} exists. The notation of equations (4.31) and (4.32) can now be simplified to

$$\underset{\sim}{\delta}^{i'} = \underset{\sim}{\Gamma}^{i}\underset{\sim}{\delta}^{i} \tag{4.35}$$

$$\underset{\sim}{f}^{i'} = \underset{\sim}{\Gamma}^{i}\underset{\sim}{f}^{i} \tag{4.36}$$

where:

$$(\underset{\sim}{\delta}^{i})^T = <\delta_1^{i}, \delta_2^{i}, \delta_3^{i}, \delta_4^{i}, \delta_5^{i}, \delta_6^{i}> \tag{4.37}$$

$$(\underset{\sim}{f}^{i})^T = <f_1^{i}, f_2^{i}, f_3^{i}, f_4^{i}, f_5^{i}, f_6^{i}> \tag{4.38}$$

E. THE ELEMENTAL SYSTEM OF EQUATIONS

Recall from equation (4.30) that $\underset{\sim}{k}^{i'}\underset{\sim}{\delta}^{i'} = \underset{\sim}{f}^{\,i'}$. Substituting equations (4.35) and (4.36) into equation (4.30) yields

$$\underset{\sim}{k}^{i'}\underset{\sim}{\Gamma}^{i}\underset{\sim}{\delta}^{i} = \underset{\sim}{\Gamma}^{i}\underset{\sim}{f}^{i} \tag{4.39}$$

Multiplying both sides of equation (4.46) by the inverse of the transformation matrix, $\underset{\sim}{\Gamma}^{i}$, yields

$$(\underset{\sim}{\Gamma}^{i})^{-1}\underset{\sim}{k}^{i'}\underset{\sim}{\Gamma}^{i}\underset{\sim}{\delta}^{i} = (\underset{\sim}{\Gamma}^{i})^{-1}\underset{\sim}{\Gamma}^{i}\underset{\sim}{f}^{i}$$

which simplifies to

$$(\underset{\sim}{\Gamma}^{i})^{-1}\underset{\sim}{k}^{i'}\underset{\sim}{\Gamma}^{i}\underset{\sim}{\delta}^{i} = \underset{\sim}{f}^{i} \tag{4.40}$$

Since $\underset{\sim}{\Gamma}^{i}$ is an orthogonal matrix, $(\underset{\sim}{\Gamma}^{i})^{-1} = (\underset{\sim}{\Gamma}^{i})^{T}$, and equation (4.41) can be rewritten as:

$$(\underset{\sim}{\Gamma}^{i})^{T}\underset{\sim}{k}^{i'}(\underset{\sim}{\Gamma}^{i})\underset{\sim}{\delta}^{i} = \underset{\sim}{f}^{i} \tag{4.41}$$

yielding the elemental system of equations transformed to the horizontal/vertical coordinate system. Now, the elemental stiffness matrices and force vectors are ready for the construction of the global stiffness matrix and force vector. The $(\underset{\sim}{\Gamma}^{i})^{T}\underset{\sim}{k}^{i'}(\underset{\sim}{\Gamma}^{i})$ term is the elemental stiffness matrix in terms of the x- and y-coordinates, and is denoted as $\underset{\sim}{k}^{i}$, that is:

$$\underset{\sim}{k}^{i} = (\underset{\sim}{\Gamma}^{i})^{T}\underset{\sim}{k}^{i'}(\underset{\sim}{\Gamma}^{i}) \tag{4.42}$$

F. THE GLOBAL SYSTEM OF EQUATIONS

With the elemental system of equations transformed into the global (horizontal/vertical) coordinate system, the global system of equations can be formed. The system stiffness matrix $\underset{\sim}{K}$ is the union of the local transformed stiffness matrices for each element, thus

$$\underset{\approx}{K} = \cup \underset{\approx}{k}^{i} \tag{4.43}$$

where:

$$\underset{\approx}{k}^{i} = (\underset{\approx}{\Gamma}^{i})^{T} \underset{\approx}{k}^{i\prime} (\underset{\approx}{\Gamma}^{i})$$

and the global (or system) force vector $\underset{\sim}{F}$ is obtained by constructing the union of the transformed local force vectors $\underset{\sim}{f}^{i}$, that is,

$$\underset{\sim}{F} = \cup \underset{\sim}{f}^{i} \tag{4.44}$$

Then the global system of equations becomes:

$$\underset{\approx}{K} \underset{\sim}{\delta} = \underset{\sim}{F} \tag{4.45}$$

Solution of the above system stiffness equations yields the system "displacements". These horizontal, vertical and rotational degrees of freedom "displacements" must be transformed back to local axial, lateral and rotational "displacements" in order to use the stress equations based upon the beam and bar equations. First the global degrees of freedom $\delta_1, \delta_2, \ldots, \delta_n$, where n=3(NEL+1), are related to the δ_i (where i=1,2,...,6) element horizontal, vertical, and rotational degrees of freedom for each element. The j^{th} degree of freedom for the i^{th} element is given by

$$\delta_j^i = \delta_k \tag{4.46}$$

where i=1,2,...,NEL, j=1,2,3, and k=3(i-1)+j. Then the axial, lateral, and rotational "displacements" for the i^{th} element at node 1 are obtained from equation (4.31) as

$$\begin{aligned}
\delta_1^{i\prime} &= \delta_1^i \cos(\alpha_i) + \delta_2^i \cos(\beta_i) \\
\delta_2^{i\prime} &= -\delta_1^i \cos(\beta_i) + \delta_2^i \cos(\alpha_i) \\
\delta_3^{i\prime} &= \delta_3^i
\end{aligned}$$

and likewise for the node 2 end.

In this manner, the stress at each end of each element can be determined. Choosing the greater of the two stresses as the governing stress of that element, the optimization analysis can be conducted for the entire structure. In this way, the width dimension b_i of each element for a minimized weight structure is obtained.

V. PROGRAM DESCRIPTION AND CAPABILITIES

Using the previous developments of Chapters II, III, and IV, a FORTRAN 77 code was written for execution on the VAX 2000 workstation. The program, named *ARCH_OPT.FOR*, was constructed to be as fully interactive with the user as possible to eliminate the need for editing and recompiling. The applicability of the code is limited by the assumptions made in Chapter II. (i.e., rectangular cross-section, linearly elastic material, etc.) As illustrated in Figure (5.1), execution of the program opens and reads an input file, *ARCH_IN.DAT*, which contains information describing the problem being investigated. The x-y coordinates of the end points of each element as well as the element orientation is determined by the subroutine *GEOMETRY*. The subroutine *OPTIMIZATION_TOOL* contains the parameter OPTDCS, the optimization decision. With OPTDCS=1, DOT is called and the weight optimum structure is determined using the provided width dimension as the starting point of the optimization process. The stress constraint is adhered to based upon the stresses calculated by the FEM analysis contained in the subroutine *ARCH_STRESS*. If no optimization is desired, i.e., OPTDCS=0, and the program computes the stress distribution based upon the input data, treating the initial geometric parameters as the actual design. With the data thus provided, the problem is solved and an output file named *ARCH_OUT.DAT* is created. The output file contains the problem parameters, the optimized design variables (width dimensions), and the value of the resulting objective function, that is, the minimum volume.

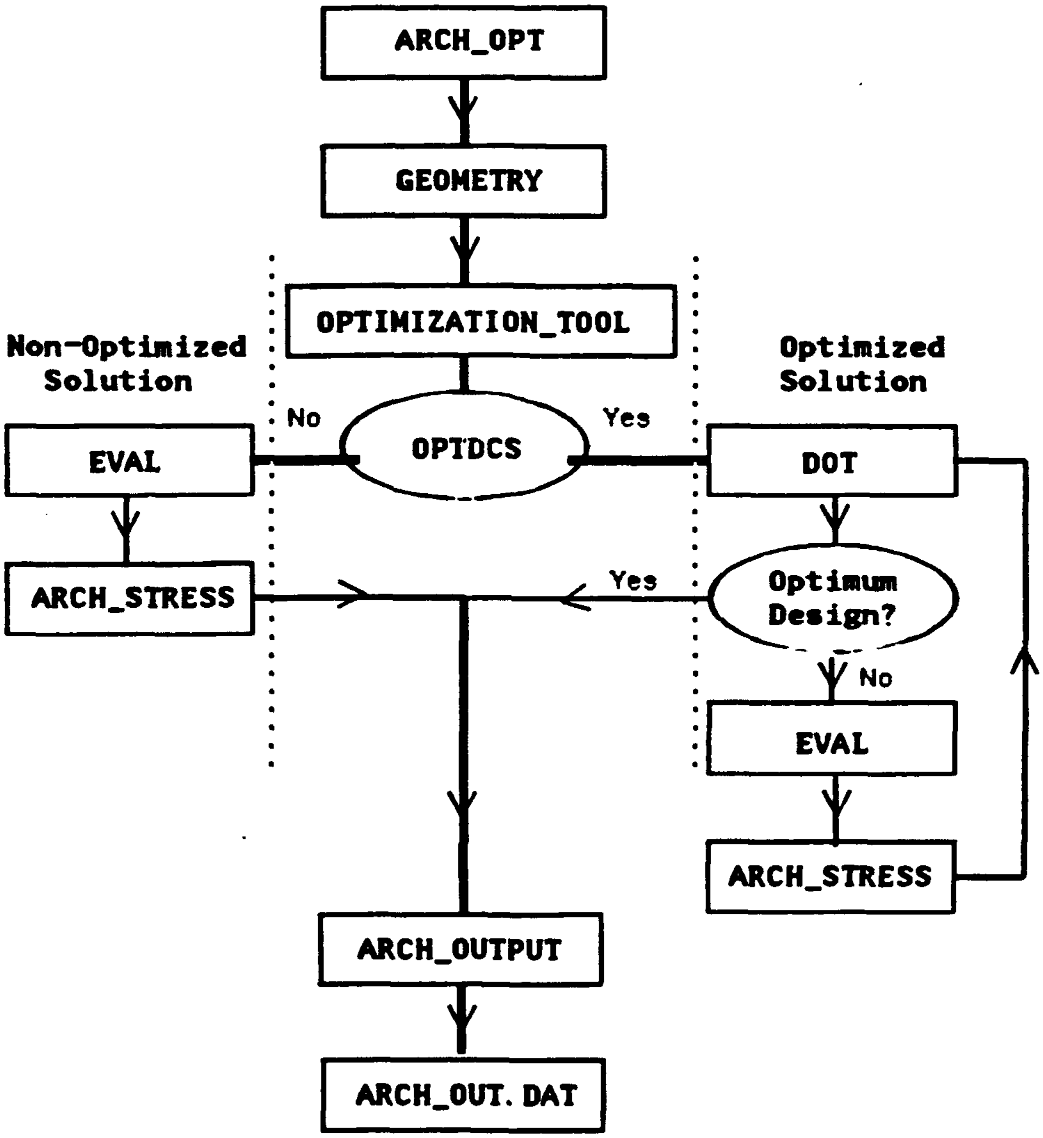


Figure 5.1: ARCH_OPT Program Structure

To better understand the program's capabilities, the data fields contained in *ARCH_IN.DAT* need to be discussed. The file is an unformatted set of twenty-five numbers separated by commas. This file must be of the form:

ANGLE, RADIUS, YOUNG, YIELD, NEL, METHOD, IPRINT, DV1BG, DV1LO, DV1UP, H, CLAN, FX, FY, FM, FA, OPTDCS, ITERATE, PRCSN, BX1, BY1, BM1, BX2, BY2, BM2

Table 5.1 describes each of these parameters. For further clarification, Figure 5.2 illustrates how the variables represent the problem and the sign conventions used.

TABLE 5.1: ARCH_IN.DAT FIELD PARAMETERS

ANGLE	A real number from 0 to 180 representing the angle subtended by the arch (in degrees).
RADIUS	A real number representing the length of the arch. (Dimensions are arbitrary, but they must remain consistent for all inputs!)
YOUNG	A real number representing the Young's Modulus of the arch material.
YIELD	A real number representing the yield strength of the material used. If a factor of safety is desired, it should first be accounted for and the resultant design strength used.
NEL	An integer value from 1 to 32 which denotes the number of elements the user wishes to divide the arch for FEM evaluation. The program is capable of up to 32 elements.
METHOD	An integer from 1 to 2. This is a parameter called by DOT to allow the user to select which optimization method is to utilized. **METHOD=1** Modified Method of Feasible Directions **METHOD=2** Sequential Linear Programming NOTE: If the problem is unconstrained, the BFGS algorithm will be used by default [Ref. 5, p. 2-5]
IPRINT	An integer from 0 to 5 used by DOT to control the output data from the DOT optimization. See Appendix C for the specific outputs
DV1BG	A real number which represents the design variable 1 (width dimension) best guess. It initializes all element width dimension to the best guess value. This establishes the optimization starting point.

DV1LO A real number which represents design variable 1's lower limit. (The lower side constraint for the width dimension)

DV1UP A real number which represents design variable 1's upper limit. (The upper side constraint for the width dimension)

H A real number which represents the constant height (depth) of the arch.

CLAN An integer which represents the number of the node at which a concentrated load is to be applied. This number must be from 1 to NEL+1. If no concentrated load is desired, FX, FY, and FM should be made to equal zero.

FX A real number which represent the magnitude of a concentrated load in the horizontal direction. FX is applied at node "CLAN".

FY A real number which represent the magnitude of a concentrated load in the vertical direction. FY is applied at node "CLAN".

FM A real number which represent the magnitude of a concentrated moment. FM is applied at node "CLAN".

FA A real number which represents the magnitude of a uniformly distributed lateral load which spans the entire length of the arch.

OPTDCS An integer value which represents the optimization "decision" such that:

OPTDCS=1 Optimize the dimension of the problem

OPTDCS=2 Do not optimize the problem. This choice will calculate the stress distribution of the arch based upon the current problem dimensions, assuming the width dimension to be constant and equal to DV1BG

ITERATE An integer value which represents the number of times the resulting "optimized" variables are to be re-entered into DOT and the optimization performed again. This technique was found to be most useful in refining the optimized solutions.

PRCSN An integer value from 1 to 2. This parameter allows the user to solve the FEM linear system of equations in :

PRCSN=1: single precision

PRCSN=2: double precision

B_1 An integer value represent the boundary condition of the arch's first nodal point such that if the value is 0, The end is free to move in that degree of freedom. If the value is 1, the end is fixed in that degree of freedom.
BX1 = horizontal displacement at point A
BY1 = vertical displacement at point A
BM1 = slope of the beam at point A

B_2 An integer value represent the boundary condition of the arch's last nodal point such that if the value is 0, The end is free to move in that degree of freedom. If the value is 1, the end is fixed in that degree of freedom.
BX2 = horizontal displacement at point C
BY2 = vertical displacement at point C
BM2 = slope of the beam at point C

In summary, for a given geometry, loading, and set of boundary conditions, the program is able to determine the optimum width dimension of each element throughout the length of the arch. This results in an arch of minimum volume (weight), capable of supporting the given loading. If desired, the optimization process can be bypassed completely. This results in the determination of the arch stress distribution based upon the input parameters, that is, the stresses associated with the initial design dimensions. These factors combine to make *ARCH_OPT* a useful tool in evaluating a variety of arches in engineering applications.

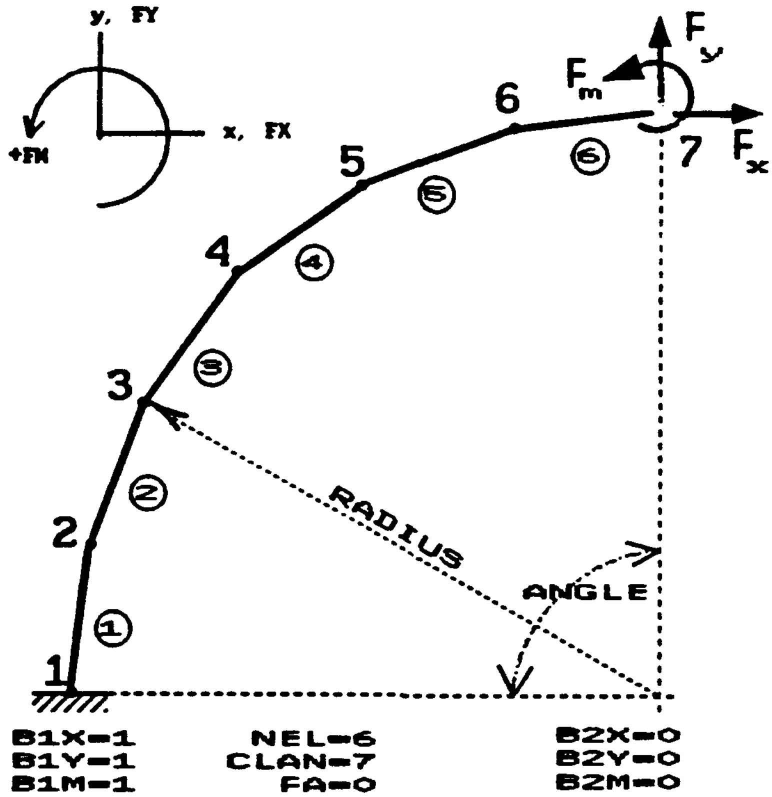


Figure 5.2: ARCH_IN.DAT Variable Implementation

VI. PROGRAM VERIFICATION

To verify the finite element code used in the optimization investigation, several non-optimum problems of beam and arch structures with known analytical solutions were solved using the code. The code solution of these problems also served the purpose of establishing a relation between the number of elements used in the model and the accuracy of the method. With a "yardstick" thus provided, "*ARCH_OPT.FOR*"'s capabilities for accurate modeling of beams and arches was assumed.

The first verification problem was a cantilever beam subjected to a concentrated end load, illustrated by Figure (6.1). Gere and Timoshenko [Ref. 6, p. 737] give general formulas for the lateral displacements and beam slopes for this case, as:

$$v = Px^2(3L-x)/6EI \qquad (6.1a)$$

$$v' = Px(2l-x)/2EI \qquad (6.1b)$$

Using the parameters:

$P = 1000$ lbf	$h = 3.0$ inches
$L = 45$ inches	$E = 30 \times 10^6$ psi
$b = 1.5$ inches	$I = bh^3/12$

the lateral displacements and beam slopes at the midpoint and free end were calculated. *ARCH_OPT* was run for this beam structure using an angle of 45.0×10^{-6} radians and a radius of 10^6 inches to approximate the straight 45 inch beam length. For a four element approximation, Table (6.1) compares the FEM solution to the analytic solution.

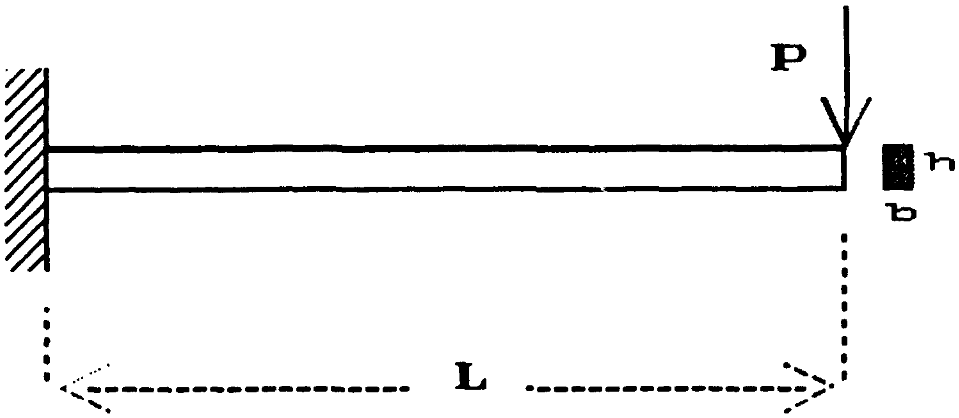


Figure 6.1: Verification Problem #1

TABLE 6.1: VERIFICATION PROBLEM #1 SUMMARY OF DISPLACEMENTS

Node	Fixed End	Mid Point	Free End
Analytic δ	0.000E+00	9.375E-02	3.000E-01
FEM δ	0.000E+00	9.375E-02	3.000E-01
% Error	fixed	0.00E+00	0.00E+00
Analytic δ'	0.000E+00	7.500E-03	1.000E-02
FEM δ'	0.000E+00	7.499E-03	9.999E-03
% Error	fixed	1.33E-02	1.00E-02
Max % Error			1.33E-02

where percent error is defined as,

$$\% \text{ Error} = 100 \times (\delta_{exact} - \delta_{FEM}) / \delta_{exact} \tag{6.2}$$

The stress corresponding to the same points of interest was calculated using equation (3.1). Recall

$$\sigma_n = Mc/I$$

which in terms of the beam equation at the i^{th} node becomes

$$(\sigma_n)_i = (EIv''h/2)/(b_ih^3/12)$$

or simply

$$(\sigma_n)_i = 6EIv''/b_ih^2 \tag{6.3}$$

From equation (4.1) of the FEM development, $v = Q^T\underset{\sim}{v}$, hence

$$v'' = (Q^T\underset{\sim}{v})''$$

$$v''(x_i) = (Q^T)''\underset{\sim}{v} = (-6/l^2)\underset{\sim}{v}_i + (4/l)\underset{\sim}{v}_i' \tag{6.4}$$

Substituting equation (6.4) into equation (6.3) yields

$$(\sigma_n)_i = (6EI/b_ih^2)[-6v_i/l^2 + 4v_i'/l] \tag{6.5}$$

where v_i and v_i' are the lateral dispiacement and slope at x_i and are obtained from $\delta_2^{i\prime}$, $\delta_3^{i\prime}$, $\delta_5^{i\prime}$ and $\delta_6^{i\prime}$ of equation (4.31). The stresses of equation (3.1) were then compared to those calculated by the *ARCH_OPT* using equation (6.5). The results, summarized in Table (6.2), show a maximum difference of 5.00×10^{-4}% between the exact and FEM solution.

TABLE 6.2: VERIFICATION PROBLEM #1 SUMMARY OF STRESSES

Verification Problem #1
(Stresses)

Node	Fixed End	Mid Point	Free End
Analytic σ	2.000E+04	1.000E+04	0.000E+00
FEM σ	1.999E+04	9.999E+03	9.712E-06
% Error	5.00E-04	5.00E-04	N/A
Max % Error			5.00E-04

The second verification problem considered was that of a prismatic bar, fixed at one end and subjected to an axial concentrated load at the free end as shown in Figure (6.2).

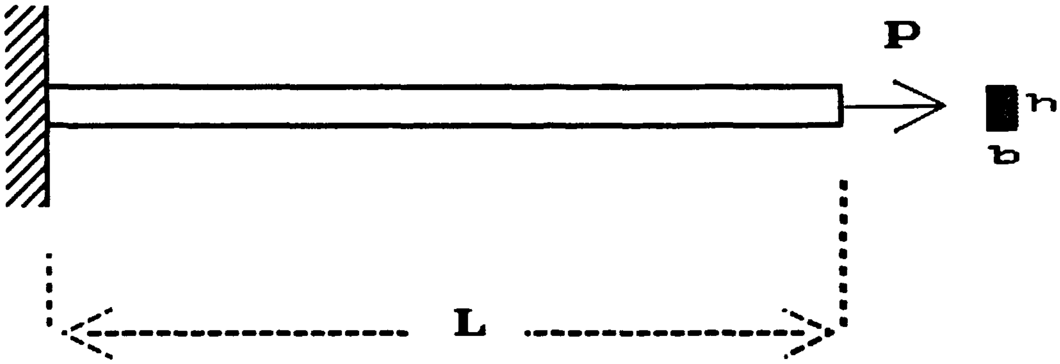


Figure 6.2: Verification Problem #2

The parameters used were:

P = 1000 lbf	h = 3.0 inches
L = 45 inches	$E = 30 \times 10^6$ psi
b = 1.5 inches	$I = bh^3/12$

From the Bar Equation (1.2), we have:

$$(AEu')' = p_x(x)$$

$$AEu' = F(x) \tag{6.6}$$

and the normal stress developed in a bar due to axial loading is:

$$\sigma_n = F(x)/A \tag{6.7}$$

Substituting equation (6.6) into equation (6.7) yields

$$\sigma_n = AEu'/A$$

or simply

$$\sigma_n = Eu' \tag{6.8}$$

Substituting equation (4.18) into equation (6.8) yields the FEM solution,

$$\sigma_n = E(\underset{\sim}{G}^T \underset{\sim}{u})' \tag{6.9}$$

From the FEM development,

$$(\underset{\sim}{G}^T\underset{\sim}{u})' = (u_i/l)$$

which when substituted into equation (6.9) yields the stress at the i^{th} node,

$$(\sigma_n)_i = Eu_i/l \tag{6.10}$$

where u_i, the axial displacement at x_i, is obtained from $\delta_1^{i\prime}$ and $\delta_4^{i\prime}$ of equation (4.31). Again using the midpoint and free end as the locations for conducting the displacement and stress analysis, the FEM code results are compared to those generated by the analytic solutions. These results are listed in Table (6.3). They show no difference between the FEM approximation and the analytic solution.

TABLE 6.3: VERIFICATION PROBLEM #2 SUMMARY OF RESULTS

Verification Problem #2
(Displacements)

Node	Fixed End	Mid Point	Free End
Analytic δ	0.000E+00	1.667E-04	3.333E-04
FEM δ	0.000E+00	1.667E-04	3.333E-04
% Error	fixed	0.00E+00	0.00E+00
Analytic δ'	0.000E+00	2.222E+02	2.222E+02
FEM δ'	0.000E+00	2.222E+02	2.222E+02
% Error	fixed	0.00E+00	0.00E+00
Max % Error			0.00E+00

The third verification problem chosen was a cantilever beam with a concentrated moment at the free end, illustrated by Figure (6.3). Again, from Gere and Timoshenko [Ref. 6, p.737], the analytic solution for the displacements and slopes of this particular problem are given by the equations:

$$v = M_o x^2/2EI \tag{6.11a}$$

$$v' = M_o x/EI \tag{6.11b}$$

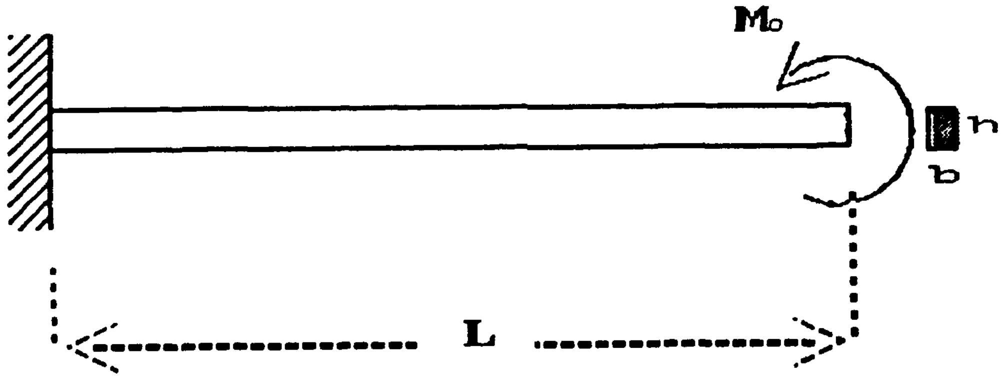


Figure 6.3: Verification Problem #3

Using the parameters:

M_o = 10,000 lbf	h = 3.0 inches
L = 45 inches	E = 30 x 10^6 psi
b = 1.5 inches	I = $bh^3/12$

the displacements and slopes were determined for the points of interest. These results are compared to the 4-element FEM approximations in Table (6.4).

TABLE 6.4: VERIFICATION PROBLEM #3 SUMMARY OF DISPLACEMENTS

Verification Problem #3
(Displacements)

Node	Fixed End	Mid Point	Free End
Analytic δ	0.000E+00	2.500E−02	1.0000E−01
FEM δ	0.000E+00	2.500E−02	9.9999E−02
% Error	fixed	0.00E+00	1.00E−03
Analytic δ'	0.000E+00	2.222E−03	4.444E−03
FEM δ'	0.000E+00	2.222E−03	4.444E−03
% Error	fixed	0.00E+00	0.00E+00
Max % Error			1.00E−03

A comparison of the FEM approximations and analytical solutions again show virtually no disparity. These results are presented in Table (6.5).

TABLE 6.5: VERIFICATION PROBLEM #3 SUMMARY OF STRESSES

Verification Problem #3 (Stresses)			
Node	Fixed End	Mid Point	Free End
Analytic σ	4.444E+03	4.444E+03	4.444E+03
FEM σ	4.444E+03	4.444E+03	4.444E+03
% Error	0.00E+00	0.00E+00	0.00E+00
Max % Error			0.00E+00

The final verification problem is based upon an example arch problem presented by Gere and Timoshenko [Ref. 6, p.616]. They demonstrate how the unit-load method can be used to calculate the horizontal displacements of the problem illustrated in Figure (6.4). The following formula was obtained:

$$\delta_h = PR^3/2EI \tag{6.12}$$

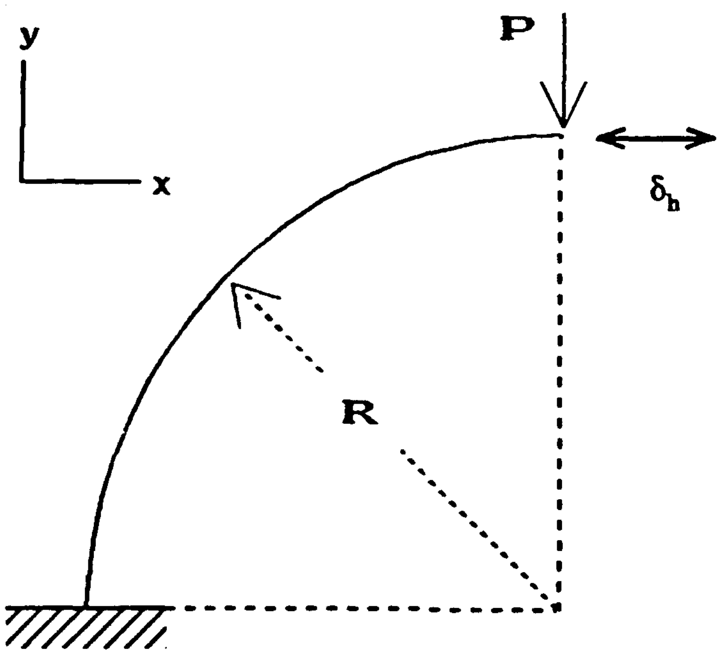


Figure 6.4: Arch Verification Problem #4

For the parameters:

$P = 1000$ lbf	$\Theta = 90°$
$R = 45$ inches	$E = 30 \times 10^6$ psi
$b = 1.5$ inches	$I = bh^3/12$
$h = 3.0$ inches	

the horizontal deflection is found to be 0.4500 inches. The horizontal deflection from the 4-element FEM approximation is 0.4470 inches, an error of 0.66%.

In all of the verification problems, the percent differences between the values obtained by the approximate FEM method and the exact solutions were in all cases less than 0.66%. Satisfied that the program was producing very good data, the investigation to obtain optimum structures was pursued.

VII CASE STUDIES

Recognizing an arch to be but another structural means of transferring a load from one point to another, the desire to compare the efficiency of the optimized arch to that of a traditional structure drove the first two cases studied. Given the problem of transferring the load at B to A, as illustrated in Figure (7.1a), numerous structures could be used. For brevity, only the frame (7.1b) and arch (7.1c) will be studied. Given the parameters:

$E = 30 \times 10^6$ psi	$h = 2$ inches
$S_y = 52{,}000$ psi	$a = 32$ inches
$I = bh^3/12$	$b = 32$ inches

only the width dimension for each case will be allowed to vary. In this way, a volume comparison of each structure, hence a measure of the relative efficiency of the structure, may be made.

For the non-optimized frame shown in Figure (7.1b), the base (or width) dimension is considered constant throughout. In order to keep the maximum stress below the yield strength of the arch material, we have

$$(\sigma_n)_{max} \leq S_y$$

or

$$M_{max} c/I \leq 52{,}000 \text{ psi} \qquad (7.1)$$

The maximum moment occurs uniformly along the vertical member of the frame, hence

$$[(2000 \text{ lbf})(32 \text{ in})(2 \text{ in}/2)]/[b(2 \text{ in})^3/12] \leq 52{,}000 \text{ psi}$$

or simply

$$1.846 \text{ in.} \leq b \tag{7.2}$$

The total volume of the frame is

$$\text{Volume} = (32 \text{ in})(2 \text{ in})(1.846 \text{ in}) + (32 \text{ in})(2 \text{ in})(1.846 \text{ in})$$

or

$$\text{Volume} = 236.3 \text{ in}^3 \tag{7.3}$$

This volume will be the basis upon which the following optimized volumes/weights hence efficiencies will be based.

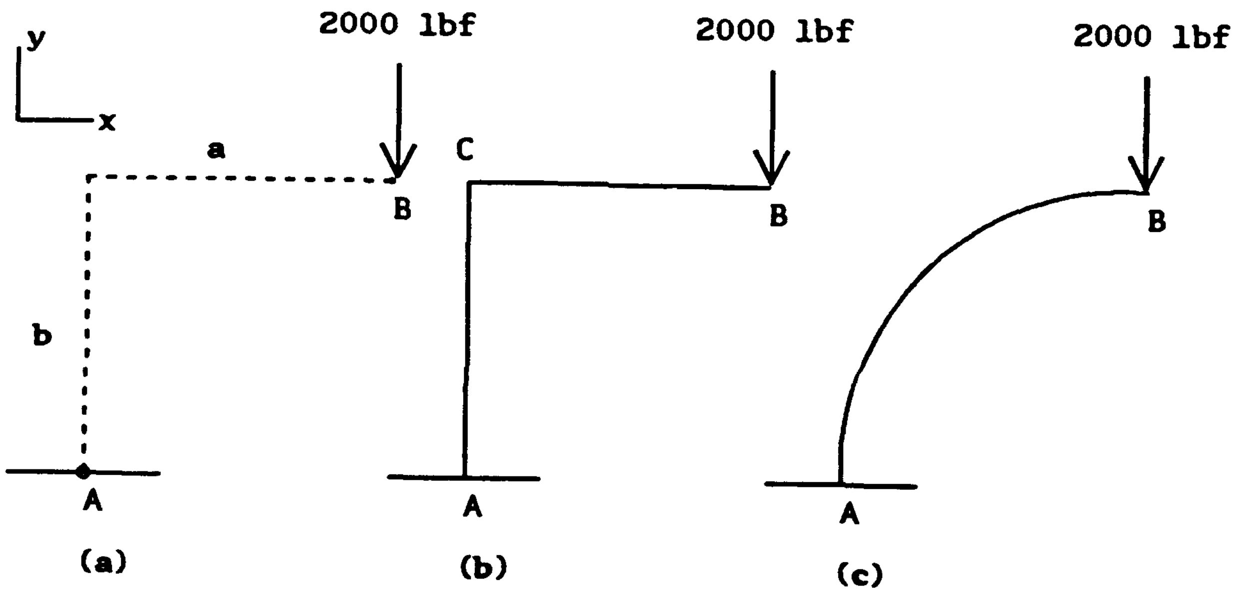


Figure 7.1: Methods of Transferring a Load

A. CASE 1: THE OPTIMIZED FRAME

Given the problem presented in Figure (7.1), an optimized frame of equal load bearing capabilities was sought. First, the frame was divided into its vertical and horizontal members. The vertical member is subjected to the concentrated moment, aP, at C, resulting in a uniform moment and hence uniform stress along the member. Consequently the vertical member has a uniform width dimension of 1.846 inches as previously determined.

The horizontal member is a cantilever beam subjected to a concentrated end load. Since the moment along BC varies linearly from 0 at B to aP at C, for a constant $\sigma_{max}=S_y$, the width must also vary linearly. Thus the width varies linearly from 0 inches at the right end to 1.846 inches at C. The volume for this frame is:

$$\text{Volume} = (32\text{ in})(2\text{ in})(1.848\text{ in}) + (.5)(32\text{ in})(2\text{ in})(1.848\text{ in})$$

which is:

$$\text{Volume} = 177.4\text{ in}^3 \tag{7.4}$$

The volume of the horizontal member, BC, was then optimized using a 4-element, 8-element, and 12-element discretization. Table (7.1) illustrates the optimized volumes of each of these solutions. The percent difference between the 4-element and 8-element solution was found to be less than 11.5% and that for the 8-element and 12-element solution to be less than 3%. In the interest of solving many cases, it was decided to solve all future case studies with a 12-element discretization, treating the 12-element model as producing grid independent results.

The optimized results for the horizontal member, given in Table (7.1) show a total member volume of 64.63 in^3. The volume of the vertical member remains the same as for the non-optimized structure, hence the total volume of the optimized frame is

$$\begin{aligned}\text{Volume} &= (32\text{ in})(2\text{ in})(1.848\text{ in}) + 64.63\text{ in}^3 \\ &= 182.9\text{ in}^3\end{aligned} \tag{7.5}$$

This represents 22.6% less volume, hence 22.6% less weight than the non-optimized frame.

TABLE 7.1: CASE 1 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in.}	Node	Stress {psi}
1	2.000E+00	2.667E+00	1.848E+00	9.856E+00	1	5.19E+04
2	2.000E+00	2.667E+00	1.695E+00	9.040E+00	2	5.19E+04
3	2.000E+00	2.667E+00	1.543E+00	8.229E+00	3	5.18E+04
4	2.000E+00	2.667E+00	1.389E+00	7.408E+00	4	5.18E+04
5	2.000E+00	2.667E+00	1.236E+00	6.592E+00	5	5.18E+04
6	2.000E+00	2.667E+00	1.082E+00	5.771E+00	6	5.18E+04
7	2.000E+00	2.667E+00	9.313E-01	4.967E+00	7	5.17E+04
8	2.000E+00	2.667E+00	7.751E-01	4.134E+00	8	5.15E+04
9	2.000E+00	2.667E+00	6.316E-01	3.369E+00	9	5.16E+04
10	2.000E+00	2.667E+00	4.676E-01	2.494E+00	10	5.07E+04
11	2.000E+00	2.667E+00	3.196E-01	1.705E+00	11	5.13E+04
12	2.000E+00	2.667E+00	2.000E-01	1.067E+00	12	5.07E+04
		12-element	Σ Volume:	6.463E+01	13	4.00E+04
		8-element	Σ Volume:	6.649E+01		
		4-element	Σ Volume:	7.385E+01		

B. CASE 2: THE OPTIMIZED CANTILEVER ARCH

Given the circumstances and parameters of Figure (7.1), a cantilever circular arch (Fig. 7.1c) was employed to perform the same function, transferring the given load at point B to point A. The resulting dimensions and stresses of the optimization are presented in Table (7.2). These results illustrate what one would have expected, the width dimension is reduced until the local stress approaches the yield strength of the material. The total volume of the arch, 128.3 in^3 is 46.3% less than the non-optimized frame and 29.9% less than the optimized frame. In moving structures where higher weights mean higher operating costs, savings such as these can become significant.

TABLE 7.2: CASE 2 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in}	Node	Stress {psi}
1	2.000E+00	4.186E+00	1.864E+00	1.561E+01	1	5.20E+04
2	2.000E+00	4.186E+00	1.842E+00	1.542E+01	2	5.20E+04
3	2.000E+00	4.186E+00	1.795E+00	1.503E+01	3	5.20E+04
4	2.000E+00	4.186E+00	1.716E+00	1.436E+01	4	5.20E+04
5	2.000E+00	4.186E+00	1.612E+00	1.350E+01	5	5.19E+04
6	2.000E+00	4.186E+00	1.477E+00	1.237E+01	6	5.19E+04
7	2.000E+00	4.186E+00	1.325E+00	1.109E+01	7	5.16E+04
8	2.000E+00	4.186E+00	1.166E+00	9.762E+00	8	5.16E+04
9	2.000E+00	4.186E+00	9.316E-01	7.799E+00	9	5.05E+04
10	2.000E+00	4.186E+00	7.126E-01	5.966E+00	10	5.19E+04
11	2.000E+00	4.186E+00	5.168E-01	4.327E+00	11	4.85E+04
12	2.000E+00	4.186E+00	3.656E-01	3.061E+00	12	3.47E+04
			Σ Volume:	1.283E+02	13	1.79E+02

C. CASE 3: THE OPTIMIZED CANTILEVER ARCH WITH AXIAL END LOAD

To appreciate how the stress distribution follows the moment distribution of the member, the problem of Case 2 was modified keeping the same parameters as outlined previously, by changing the direction of the load as shown in Figure (7.2).

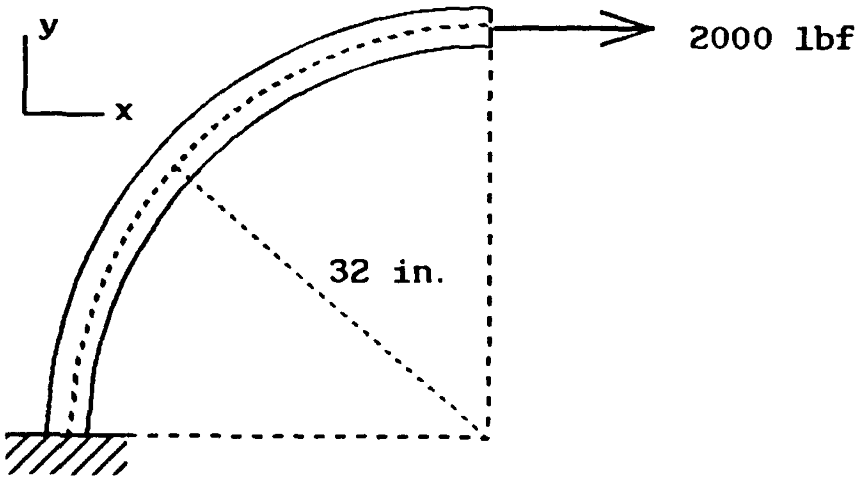


Figure 7.2: Case 3 Problem Geometry

For this case, the moment at any point is given by the expression

$$M = PR(1-\sin(\Theta)) \tag{7.6}$$

as opposed to the moment distribution of Case 2 where the moment along the length of the arch is

$$M = PR\cos(\Theta) \tag{7.7}$$

Assuming a unit moment, that is PR=1, then from Figure (7.3), one may see that the area under Case 3's moment curve is significantly less than the area under Case 2's moment curve. One would expect this to correspond with the need for less material due to less applied force (in this case, moment).

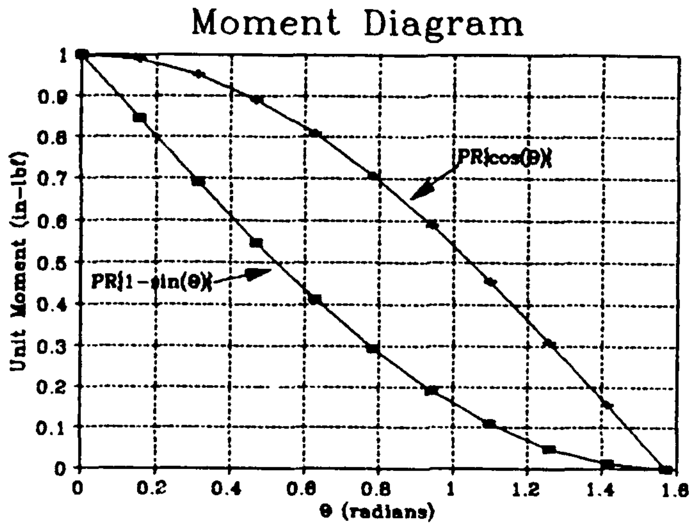


Figure 7.3: Moment Diagrams of Case 2 and Case 3

The resulting optimized arch of this case has the dimensions outlined in Table (7.3) and a noticeably smaller volume of 81.39 in^3 than the 128.3 in^3 of Case 2. In fact, the reduction in volume is the same as the reduction in areas of the moment diagrams. This result collaborates with the previous observation. Hence one may see that the normal stress is predominantly due to bending and essentially follows the moment distribution of the structure in question.

TABLE 7.3: CASE 3 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in.}	Node	Stress {psi}
1	2.000E+00	4.186E+00	1.846E+00	1.545E+01	1	5.20E+04
2	2.000E+00	4.186E+00	1.687E+00	1.412E+01	2	5.19E+04
3	2.000E+00	4.186E+00	1.435E+00	1.201E+01	3	5.20E+04
4	2.000E+00	4.186E+00	1.196E+00	1.001E+01	4	5.19E+04
5	2.000E+00	4.186E+00	9.703E-01	8.123E+00	5	5.19E+04
6	2.000E+00	4.186E+00	7.574E-01	6.341E+00	6	5.19E+04
7	2.000E+00	4.186E+00	5.676E-01	4.752E+00	7	5.19E+04
8	2.000E+00	4.186E+00	4.009E-01	3.356E+00	8	5.19E+04
9	2.000E+00	4.186E+00	2.611E-01	2.186E+00	9	5.16E+04
10	2.000E+00	4.186E+00	2.000E-01	1.674E+00	10	4.75E+04
11	2.000E+00	4.186E+00	2.000E-01	1.674E+00	11	2.13E+04
12	2.000E+00	4.186E+00	2.000E-01	1.674E+00	12	9.10E+03
			Σ Volume:	8.139E+01	13	5.00E+03

D. CASE 4: THE CANTILEVER ARCH UNDER A DISTRIBUTED LOAD

This case involves applying a uniformly distributed load acting radially inward on a cantilever arched segment as pictured in Figure (7.4)

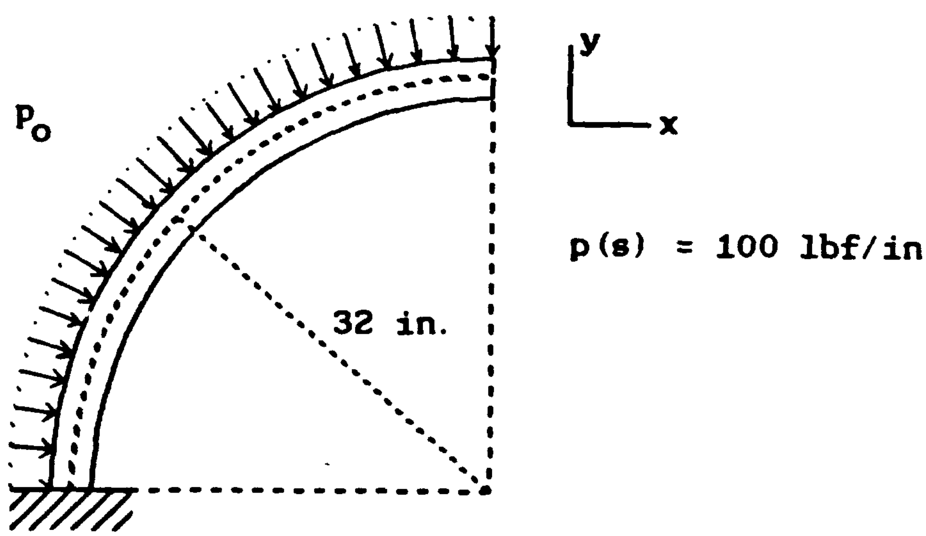


Figure 7.4: Case 4 Problem Geometry

where:

$E = 30x10^6$ psi
$S_y = 52,000$ psi
$I = bh^3/12$

h = 2 inches
R = 32 inches
Θ = 90 degrees

The results of the optimization contained in Table (7.4) illustrate how the critical constraints shift from the maximum allowable stress to the minimum allowable width dimension. Even though the moment in the structure has diminished and the stress no longer approaches the yield strength of the material, the geometric constraint prevents the cross-section from becoming so thin that the bar/beam assumption is no longer valid. The variation in the width dimensions appears to be almost logarithmic alluding to the complexities involved with arched segments under uniformly distributed loads. Had the arch not been optimized and assuming the maximum width dimension of Table (7.4) to represent the uniform width dimension of the non-optimum arch, such that

$$\text{Volume} = [(R_o)^2 - (R_i)^2](\Theta \text{ radians})(b_{max})/2 \tag{7.8}$$

then the total volume of the non-optimum arch would have been

$$\text{Volume} = 209.1 \text{ in.}^3$$

Hence, the optimized volume of 101.6 in^3 is 51% smaller than that of the non-optimized arch.

TABLE 7.4: CASE 4 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in.}	Node	Stress {psi}
1	2.000E+00	4.186E+00	2.080E+00	1.741E+01	1	5.21E+04
2	2.000E+00	4.186E+00	1.930E+00	1.616E+01	2	5.20E+04
3	2.000E+00	4.186E+00	1.762E+00	1.475E+01	3	5.20E+04
4	2.000E+00	4.186E+00	1.552E+00	1.299E+01	4	5.20E+04
5	2.000E+00	4.186E+00	1.315E+00	1.101E+01	5	5.20E+04
6	2.000E+00	4.186E+00	1.067E+00	8.933E+00	6	5.20E+04
7	2.000E+00	4.186E+00	8.213E-01	6.876E+00	7	5.20E+04
8	2.000E+00	4.186E+00	5.912E-01	4.950E+00	8	5.19E+04
9	2.000E+00	4.186E+00	3.901E-01	3.266E+00	9	5.16E+04
10	2.000E+00	4.186E+00	2.218E-01	1.857E+00	10	5.16E+04
11	2.000E+00	4.186E+00	2.000E-01	1.674E+00	11	2.52E+04
12	2.000E+00	4.186E+00	2.000E-01	1.674E+00	12	5.62E+03
			Σ Volume:	1.016E+02	13	1.10E+03

E. CASE 5: THE SIMPLY SUPPORTED ARCH

This case involved the optimization of a simply supported arch subjected to a lateral load at the midpoint, illustrated in Figure (7.5). The arch is a first order statically indeterminate structure subject to the following parameters:

$E = 30 \times 10^6$ psi $\qquad h = 2$ inches

$S_y = 52{,}000$ psi $\qquad R = 32$ inches

$I = bh^3/12 \qquad \Theta = 180$ degrees

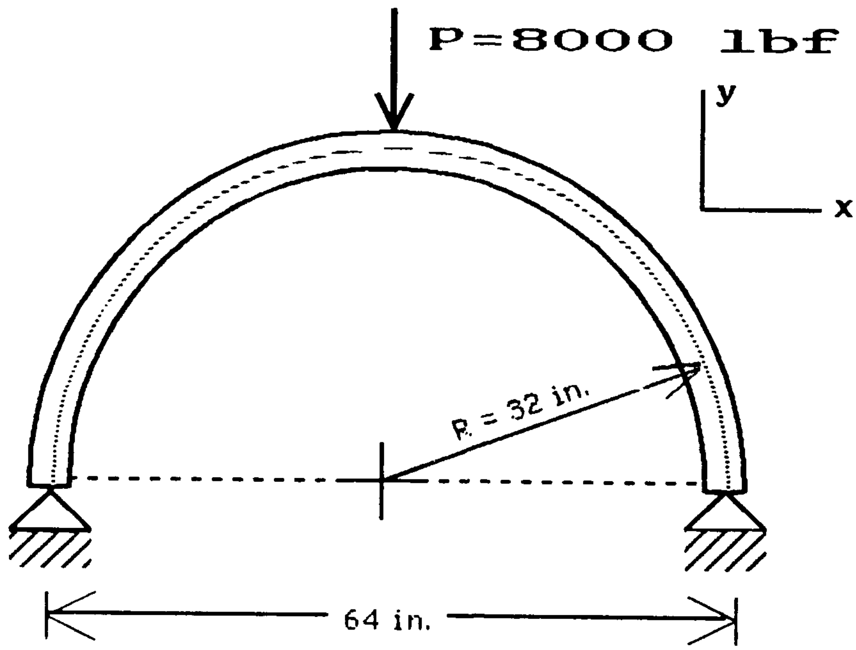


Figure 7.5: Case 5 Problem Geometry

In order to take advantage of symmetry, the problem was divided along the axis of symmetry with the following boundary conditions imposed on the symmetry end,

$$(EIv'')' = P/2$$

$$EIv' = 0$$

$$u = 0$$

The results of the optimization are summarized in Table (7.5). Here again, the weight savings of the optimized arch of 263.2 in^3 over that of the non-optimized arch, as defined by equation (7.8), of 628.1 in^3 is approximately 58%.

TABLE 7.5: CASE 5 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in.}	Node	Stress {psi}
1	2.000E+00	4.186E+00	5.130E–01	4.295E+00	1	8.08E+03
2	2.000E+00	4.186E+00	9.069E–01	7.592E+00	2	5.20E+04
3	2.000E+00	4.186E+00	1.117E+00	9.351E+00	3	5.09E+04
4	2.000E+00	4.186E+00	1.479E+00	1.238E+01	4	5.20E+04
5	2.000E+00	4.186E+00	1.196E+00	1.001E+01	5	5.20E+04
6	2.000E+00	4.186E+00	1.127E+00	9.435E+00	6	5.20E+04
7	2.000E+00	4.186E+00	1.531E+00	1.282E+01	7	4.22E+04
8	2.000E+00	4.186E+00	5.765E–01	4.826E+00	8	4.91E+04
9	2.000E+00	4.186E+00	5.935E–01	4.969E+00	9	4.93E+03
10	2.000E+00	4.186E+00	1.346E+00	1.127E+01	10	5.20E+04
11	2.000E+00	4.186E+00	2.206E+00	1.847E+01	11	5.20E+04
12	2.000E+00	4.186E+00	3.124E+00	2.615E+01	12	5.20E+04
			Σ Volume:	1.316E+02	13	5.20E+04

F. CASE 6: THE FIXED-FIXED ARCH

To determine the effect of additional redundancy, the next case involved the optimization of a fixed-fixed arch subjected to a lateral load at the midpoint, illustrated in Figure (7.6) and subject to the following parameters:

$E = 30 \times 10^6$ psi $h = 2$ inches
$S_y = 52{,}000$ psi $R = 32$ inches
$I = bh^3/12$ $\Theta = 180$ degrees

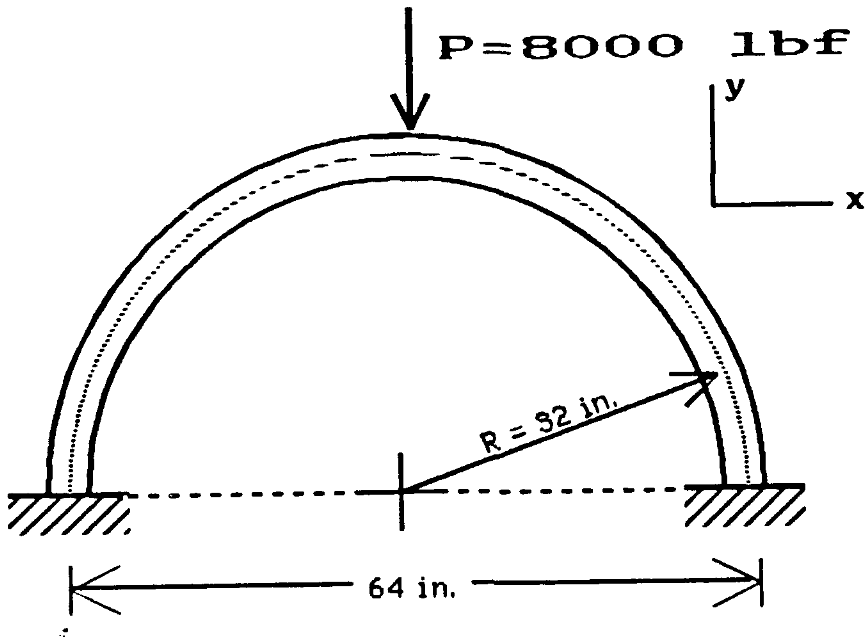


Figure 7.6: Case 6 Problem Geometry

The *fixed supports* add *two* additional redundant moments at the supports when compared to the previous simply-supported, simply-supported arch of case 5. Again taking advantage of symmetry, the problem was divided along the axis of symmetry with the following boundary conditions imposed on the symmetry end,

$$(EIv'')' = P/2$$

$$EIv' = 0$$

$$u = 0$$

The resulting optimized arch has a total volume of 210.3 in^3 as summarized in Table (7.6). The non-optimum arch has a volume of 533.0 in^3 assuming a constant arch width corresponding to the optimized arch's maximum width of 2.651 inches. Here, the weight savings of the optimized arch over that of the non-optimized arch is approximately 60%. It is also *worth particular note that though the loading and geometry of* Cases 5 and 6 are the same, differing only with regards to the boundary conditions imposed, Case 6 is 20% lighter. This is because a fixed-fixed structure is more statically indeterminate than the simply supported structure. Hence we see here that the more redundant structure is the more efficient member. This illustrates one of the reasons why fixed-fixed structures are preferred in construction.

TABLE 7.6: CASE 6 SUMMARY OF RESULTS

Element	Height {inches}	Length {inches}	Base {inches}	Volume {cubic in.}	Node	Stress {psi}
1	2.000E+00	4.186E+00	1.727E+00	1.446E+01	1	5.20E+04
2	2.000E+00	4.186E+00	9.458E-01	7.918E+00	2	5.00E+04
3	2.000E+00	4.186E+00	2.719E-01	2.276E+00	3	4.62E+04
4	2.000E+00	4.186E+00	6.504E-01	5.445E+00	4	5.20E+04
5	2.000E+00	4.186E+00	1.091E+00	9.133E+00	5	4.94E+04
6	2.000E+00	4.186E+00	7.899E-01	6.613E+00	6	5.20E+04
7	2.000E+00	4.186E+00	1.062E+00	8.891E+00	7	5.18E+04
8	2.000E+00	4.186E+00	7.539E-01	6.311E+00	8	4.21E+04
9	2.000E+00	4.186E+00	2.585E-01	2.164E+00	9	5.20E+04
10	2.000E+00	4.186E+00	9.328E-01	7.809E+00	10	5.13E+04
11	2.000E+00	4.186E+00	1.753E+00	1.468E+01	11	5.24E+04
12	2.000E+00	4.186E+00	2.651E+00	2.219E+01	12	5.19E+04
			Σ Volume:	1.079E+02	13	5.19E+04

VIII CONCLUSIONS

The conclusions of this study are:

- The stress analysis based upon the bar/beam model yielded good results with percent deviations from known analytic solutions ranging between 0.1 and 1.5%. Hence, the bar/beam element model is a viable technique in the approximation of arch structures.

- The DOT optimization software was able to utilize the bar/beam modeled stress analysis to efficiently determine weight optimum arch structures.

- The optimization demonstrates how structures which are more statically indeterminate (redundant) are likewise more efficient than identical structures under identical loading.

- The weight optimization of a structure is available and effective for all types of problem boundary conditions.

It should be noted once again that this is an initial investigation into the weight optimization of arches. Numerous opportunities exist for the expansion of the basic assumptions made in this study. This investigation only considered the optimization of structures of the form:

$$I = kA$$

The general form of this type of optimization is such that

$$I = kA^n$$

where n = 1, 2, or 3 depending upon which cross-sectional dimension(s) is defined as the design variable. Some of the possibilities for future research include:

- Allowing only the cross-sectional height dimension, h, vary while holding all other parameters constant. (n=3)

- Allowing both the height and width dimensions to vary proportionally, while holding all other parameters constant. (n=2)

- Allowing the radius of curvature of the arch (its center-line shape) to vary.

- Optimizing the arch using engineering cross-sections such as box beams, I-beams, circular cross-section, etc.

- Incorporating additional constraints (such as arch maximum height limitations, buckling constraints, crippling constraints, etc,) in order to expand the model; thereby enabling the model to solve a greater variety of problems.

APPENDIX A

JUSTIFICATION FOR OMITTING SHEAR STRESSES

The shear stress distribution through a beam of rectangular cross-section has a parabolic distribution along the height of the member. The maximum shear stress, located at the neutral axis of the beam, is

$$\tau_{max} = 1.5V/A \tag{A.1}$$

where τ_{max} is the maximum shear stress, V is the shear force, and A is the cross-sectional area of the beam. [Ref. 6, p. 229]

The normal stress due to bending is given by the equation

$$\sigma_n = Mc/I \tag{A.2}$$

where σ_n is the maximum normal stress, M is the bending moment, and I is the cross-sectional moment of inertia which for this case is $bh^3/12$ where b and h are the width and height respectively of the cross-section.

Redefining the normal stress in terms of the cross-sectional dimensions yields

$$\sigma_n = M(h/2)/(bh^3/12)$$

or

$$\sigma_n = 6M/hA \tag{A.3}$$

The ratio of the maximum shear stress to the normal stress due to bending, is denoted by r and given by the expression:

$$r = \tau_{max}/\sigma_n \tag{A.4}$$

Substituting equations (A.1) and (A.3) into equation (A.4) yields

$$r = (1.5V/A)/(6M/hA)$$

or

$$r = Vh/4M \tag{A.5}$$

For the cases investigated in this study, the maximum value r can attain is when the loading is that of a uniformly distributed load, p_y. Then, where:

$$V = p_yL \tag{A.6}$$

$$M = p_yL^2/2 \tag{A.7}$$

which upon substitution into equation (A.8) yields

$$r = (p_yL)h/4(p_yL^2/2)$$

which simplifies to

$$r = h/2L \tag{A.8}$$

The use of the beam equation requires the length of the beam to be at a minimum ten times the height, that is:

$$L \geq 10h \tag{A.9}$$

To maximize the value of r, let L equal 10h, the minimum allowable length. Substituting this value of L into equation (A.8) yields

$$r \leq h/2(10h)$$

or simply

$$r \leq 1/20 \tag{A.10}$$

Hence, the maximum shear stress accounts for less than five percent of the bending stress developed in the structure. Five percent is high considering this analysis over-assumed the value of the shear stress by assigning the maximum shear stress to the entire cross-section of the beam. Moreover, at the outermost fibers where σ_n is a maximum, the shear stress is zero.

Therefore, under the circumstances of this study, the addition of shear stresses was deemed to be unwarranted.

APPENDIX B

DOT USERS MANUEL, SECTION 2.1

DOT WITH APPLICATION PROGRAMS

2.1 CALLING STATEMENT

DOT is invoked by the following FORTRAN calling statement in the user's program:

```
CALL DOT ( INFO, METHOD, IPRINT, NDV, NCON, X,
* XL, XU, OBJ, MINMAX, G, RPRM, IPRM, WK, NRWK,
* IWK, NRIWK)
```

APPENDIX C

DOT USERS MANUEL, SECTION 2.2

2.2 PARAMETERS IN THE CALLING STATEMENT

Table 2-1 lists the parameters in the calling statement to DOT. Where arrays are defined, the required dimension size is given as the array argument. These are minimum dimensions. The arrays can be dimensioned larger than this to allow for program expansion.

TABLE 2-1: PARAMETERS IN THE DOT ARGUMENT LIST

PARAMETER	DEFINITION
INFO	Information parameter. Before calling DOT the first time, set INFO=0. When control returns from DOT to the calling program, INFO will normally have a value of 0 or 1. If INFO= 0, the optimization is complete (or terminated with an error message). If INFO= 1, the user must evaluate the objective, OBJ, and constraint functions, G(I), i=1,NCON, and call DOT again. A third possibility, INFO= 2, exists also. In this case, the user must provide gradient information. This is an advanced feature and is described in Section 3.2.
METHOD	Optimization method to be used. METHOD = 0 or 1 means use the modified method of feasible directions. METHOD = 2 means use the sequential linear programming method. If the problem in unconstrained (NCON = 0) the BFGS algorithm will be used, regardless of the value of the parameter METHOD.

IPRINT	Print control parameter. IPRINT = 0 no output. IPRINT = 1 internal parameters, initial information and results. IPRINT = 2 same plus objective function and X-vector at each iteration. IPRINT = 3 same plus G-vector and critical constraint numbers. IPRINT = 4 same plus gradients. IPRINT = 5 same plus search direction. NOTE: The IPRM Array contains additional print options.
NDV	Number of decision/design variables contained in vector X. NDV is the same as N in the mathematical problem statement given in Section 1.7.
NCON	Number of constraint values contained in array G. NCON is the same as M in the mathematical problem statement given in Section 1.7. NCON=0 is allowed.
X(NDV)	Vector containing the design variables. On the first call to DOT, this is the user's best guess of the design. On the final return from DOT (INFO=0 is returned), the vector X contains the optimum design.
XL(NDV)	Array containing lower bounds on the design variables, X. If no lower bounds are imposed on one or more of the design variables, the corresponding component(s) of XL must be set to a large negative number, say -1.0E+15. Be sure it's -1.0E+15 and not -1.0E-15 (+15, not -15).

XU(NDV)	Array containing upper bounds on the design variables, X. If no upper bounds are imposed on one or more of the design variables, the corresponding component(s) of XU must be set to a large positive number, say 1.0 E+15.
OBJ	Value of the objective function corresponding to the current values of the design variables contained in X. On the first call to DOT, OBJ need not be defined. DOT will return a value of INFO=1 to indicate that the user must evaluate OBJ and call DOT again. Subsequently, any time a value of INFO=1 is returned from DOT, the objective, OBJ, must be evaluated for the current design and DOT must be called again. OBJ has the same meaning as F(X) in the mathematical problem statement given in Section 1.7.
MINMAX	Integer parameter specifying whether the minimum (MINMAX=0,-1) or maximum (MINMAX=1) of the objective function is to be found.
G(NCON)	Array containing the NCON inequality constraint values corresponding to the current design contained in X. On the first call to DOT, the constraint values need not be defined. On return from DOT, if INFO=1, the constraints must be evaluated for the current X and DOT must be called again. If NCON=0, array G must be dimensioned to 1 or larger, but no constraint values need to be provided.

RPRM(20)	Array containing the real (floating point numbers) control parameters. Initialize the entire array to 0.0 to use all default values. If you use other values than the defaults, set the corresponding entries to the desired values. Section 3.1 describes how to change the value of these parameters.
IPRM(20)	Array containing the integer control parameters. As with the RPRM array, set the array to zero to use the default values, or set the proper entries to the desired values. Section 3.1 describes how to change the value of these parameters.
WK(NRWK)	User provided work array for real (floating point) variables. Array WK is used to store internal scalar variables and arrays used by DOT. If the user has not provided enough storage, DOT will print the appropriate message and terminate the optimization.
NRWK	Dimensioned size of work array WK. NRWK should be set quite large, starting at about 500 for a small problem. If NRWK has been given too small a value, an error message will be printed and the optimization will be terminated.
IWK(NRIWK)	User provided work array for integer (fixed point) variables. Array IWK is used to store internal scalar variables and arrays used by DOT. If the user has not provided enough storage, DOT will print the appropriate message and terminate the optimization.

NRIWK	Dimensioned size of work array IWK. A good estimate is 300 for a small problem. Increase the size of NRIWK as the problem grows larger. If NRIWK is too small, an error message will be printed and the optimization will be terminated.

Note: The minimum required values of NRWK and NRIWK are defined as follows (The dimensions may be larger than this):

N1 = NCON + NDV

N2 = 2*NDV

N3 = 10*NDV

N4 = MIN (N1,N2)

N5=1

IF NCON = 0, N5=0

NCOLA = MAX (N3,N4)

NGMAX = MIN (NCON,NCOLA)

NRB = MIN (N1,NCOLA+1)

IF NCON = 0, NRB = 1

NRWK = NDV*(10+NCOLA) + 5*NCON + NCOLA + NRB**2 +
MAX(NDV,NCOLA)+2*N5*NRB + 40

IF METHOD > 1, NRWK = NRWK + NGMAX +
NDV*(3+NCOLA)

NRIWK = NDV + NCON + NGMAX + 71

IF METHOD > 1, NRIWK = NRIWK + NGMAX

A program called DTSTOR is provided with DOT. If you compile and run this program interactively, the minimum required values of NRWK and NRIWK are calculated for you. See Appendix B for more information on this option.

APPENDIX D

VERIFICATION AND CASE STUDY OUTPUT

```
VERIFICATION #1

 OPTIMIZATION SOLUTION
 ----------------------------------------------------------------------
 A) Problem Parameters:
    Arch Angle :        0.003     Youngs Modulus:  30000000.0
    Arch Radius: 1000000.000     Yield Strength:      52000.0
    Arch Height:        3.000     No of Elements:          4

 B) Derived Constants:
    No of System Nodal Points...     5
    No of Degrees of Freedom....    15
    Length per Element..........     11.2500
    Number of Iterations........     0

 C) Structure Loading:
   FX.........................   1000.0000
   FY.........................      0.0000
   FM.........................      0.0000
   FA.........................      0.0000

 D) Elemental Dimensions and Stress Distribution:
   Element     Height          Base          Length       Volume
      1       3.00000        1.50000       11.24996     50.62481
      2       3.00000        1.50000       11.24996     50.62481
      3       3.00000        1.50000       11.24996     50.62481
      4       3.00000        1.50000       11.24996     50.62481

 E) Objective Function:
    Total structure Volume:  202.499222
   Node       Stress
      1   19999.90
      2   14999.94
      3   9999.967
      4   4999.973
      5  9.7119728E-06

 F) Boundary Conditions:
   Node     X-Displ        Y-Displ         Slope
      1           1              1             1
      5           0              0             0

 G) Solution Vector:
   Node     X-Displ        Y-Displ         Slope
      1  0.000000E+00  0.000000E+00  0.000000E+00
      2  0.257809E-01  0.112328E-08 -0.437496E-02
      3  0.937488E-01  0.409060E-08 -0.749993E-02
      4  0.189841E+00  0.828730E-08 -0.937491E-02
      5  0.299996E+00  0.130987E-07 -0.999991E-02
```

VERIFICATION #2

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.003	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
Arch Height:	3.000	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	11.2500
Number of Iterations........	0

C) Structure Loading:

FX..........................	0.0000
FY..........................	1000.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	3.00000	1.50000	11.24996	50.62481
2	3.00000	1.50000	11.24996	50.62481
3	3.00000	1.50000	11.24996	50.62481
4	3.00000	1.50000	11.24996	50.62481

E) Objective Function:

Total structure Volume: 202.499222

Node	Stress
1	222.2231
2	222.2229
3	222.2227
4	222.2224
5	222.2222

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.112328E-08	0.833330E-04	-0.191236E-09
3	0.409060E-08	0.166666E-03	-0.327832E-09
4	0.828730E-08	0.249999E-03	-0.409790E-09
5	0.130987E-07	0.333332E-03	-0.437110E-09

VERIFICATION #3

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.003	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
Arch Height:	3.000	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	11.2500
Number of Iterations........	0

C) Structure Loading:

FX.........................	0.0000
FY.........................	0.0000
FM.........................	10000.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	3.00000	1.50000	11.24996	50.62481
2	3.00000	1.50000	11.24996	50.62481
3	3.00000	1.50000	11.24996	50.62481
4	3.00000	1.50000	11.24996	50.62481

E) Objective Function:

Total structure Volume: 202.499222

Node	Stress
1	4444.438
2	4444.438
3	4444.441
4	4444.441
5	4444.441

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.624994E-02	-0.273194E-09	0.111110E-02
3	-0.249998E-01	-0.109277E-08	0.222221E-02
4	-0.562495E-01	-0.245874E-08	0.333332E-02
5	-0.999991E-01	-0.437110E-08	0.444442E-02

VERIFICATION #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	45.000	Yield Strength:	52000.0
Arch Height:	3.000	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	17.5581
Number of Iterations........	0

C) Structure Loading:

FX..........................	0.0000
FY..........................	1000.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	3.00000	1.50000	17.55813	79.01159
2	3.00000	1.50000	17.55813	79.01159
3	3.00000	1.50000	17.55813	79.01159
4	3.00000	1.50000	17.55813	79.01159

E) Objective Function:

Total structure Volume: 316.046356

Node	Stress
1	20217.61
2	18601.18
3	14266.01
4	7777.182
5	43.39704

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.654617E-01	0.131512E-01	0.750656E-02
3	-0.223502E+00	0.118880E+00	0.138705E-01
4	-0.381543E+00	0.355536E+00	0.181227E-01
5	-0.447005E+00	0.684770E+00	0.196159E-01

OPTIMIZATION #1

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.002	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	4

B) Derived Constants:

No of System Nodal Points...	5
No of Degrees of Freedom....	15
Length per Element..........	8.0000
Number of Iterations........	0

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.84550	8.00000	29.52806
2	2.00000	1.38407	8.00000	22.14510
3	2.00000	0.92261	8.00000	14.76169
4	2.00000	0.46154	8.00000	7.38456

E) Objective Function:

Total structure Volume: 73.819420

Node	Stress
1	52018.20
2	52020.44
3	52026.59
4	52000.27
5	9.4704286E-05

F) Boundary Conditions:

Node	X-Displ	Displ	Slope
1	1	1	1
5	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.508622E-01	0.221694E-08	-0.121376E-01
3	0.197286E+00	0.860890E-08	-0.236977E-01
4	0.433113E+00	0.189046E-07	-0.341030E-01
5	0.742914E+00	0.324212E-07	-0.410363E-01

OPTIMIZATION #1

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.002	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	8

B) Derived Constants:

No of System Nodal Points...	9
No of Degrees of Freedom....	27
Length per Element..........	4.0000
Number of Iterations........	1

C) Structure Loading:

FX..........................	2000.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.84548	4.00000	14.76385
2	2.00000	1.61572	4.00000	12.92572
3	2.00000	1.38417	4.00000	11.07339
4	2.00000	1.15351	4.00000	9.22805
5	2.00000	0.92224	4.00000	7.37795
6	2.00000	0.69289	4.00000	5.54310
7	2.00000	0.46127	4.00000	3.69015
8	2.00000	0.23665	4.00000	1.89324

E) Objective Function:

Total structure Volume: 66.495445

Node	Stress
1	52015.55
2	51986.30
3	52013.96
4	52013.29
5	52045.58
6	51955.51
7	52029.16
8	50706.71
9	0.2237021

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
9	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope
1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.132929E-01	0.577893E-09	-0.650196E-02
3	0.525036E-01	0.228824E-08	-0.129384E-01
4	0.117357E+00	0.511886E-08	-0.192957E-01
5	0.207485E+00	0.905344E-08	-0.255373E-01
6	0.322357E+00	0.140683E-07	-0.316093E-01
7	0.461109E+00	0.201250E-07	-0.373821E-01
8	0.622199E+00	0.271538E-07	-0.425851E-01

```
9  0.801554E+00  0.349690E-07 -0.459655E-01
```

OPTIMIZATION #1

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	0.002	Youngs Modulus:	30000000.0
Arch Radius:	1000000.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	2.6667
Number of Iterations........	4

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.84829	2.66667	9.85756
2	2.00000	1.69537	2.66667	9.04196
3	2.00000	1.54275	2.66667	8.22798
4	2.00000	1.38887	2.66667	7.40732
5	2.00000	1.23566	2.66667	6.59016
6	2.00000	1.08215	2.66667	5.77148
7	2.00000	0.93133	2.66667	4.96712
8	2.00000	0.77510	2.66667	4.13389
9	2.00000	0.63162	2.66667	3.36863
10	2.00000	0.46759	2.66667	2.49383
11	2.00000	0.31955	2.66667	1.70426
12	2.00000	0.20000	2.66667	1.06667

E) Objective Function:

Total structure Volume: 64.630867

Node	Stress
1	51925.73
2	51892.89
3	51843.36
4	51329.66
5	51784.52
6	51739.70
7	51531.16
8	51598.32
9	50656.20
10	51320.27
11	50069.02
12	39998.66
13	0.6707708

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.598324E-02	0.259434E-09	-0.442333E-02
3	0.237427E-01	0.103343E-08	-0.882640E-02
4	0.532194E-01	0.231938E-08	-0.132043E-01
5	0.943462E-01	0.411430E-08	-0.175555E-01
6	0.147043E+00	0.641459E-08	-0.218709E-01
7	0.211205E+00	0.921563E-08	-0.261415E-01
8	0.286684E+00	0.125107E-07	-0.303404E-01
9	0.373299E+00	0.162918E-07	-0.344682E-01
10	0.470718E+00	0.205440E-07	-0.384082E-01
11	0.578546E+00	0.252490E-07	-0.422098E-01
12	0.696051E+00	0.303731E-07	-0.455477E-01
13	0.820672E+00	0.358010E-07	-0.473255E-01

OPTIMIZATION #2

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loadihg:

FX.........................	0.0000
FY.........................	-2000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.86405	4.18580	15.60506
2	2.00000	1.84215	4.18580	15.42178
3	2.00000	1.79464	4.18580	15.02403
4	2.00000	1.71836	4.18580	14.38546
5	2.00000	1.61213	4.18580	13.49611
6	2.00000	1.47665	4.18580	12.36194
7	2.00000	1.32535	4.18580	11.09533
8	2.00000	1.16593	4.18580	9.76072
9	2.00000	0.93161	4.18580	7.79905
10	2.00000	0.71259	4.18580	5.96554
11	2.00000	0.51683	4.18580	4.32666
12	2.00000	0.36557	4.18580	3.06044

E) Objective Function:

Total structure Volume: 128.302124

Node	Stress
1	51975.23
2	51991.84
3	52001.67
4	51952.61
5	51914.72
6	51926.68
7	51573.75
8	50482.76
9	51888.91
10	51924.89
11	48451.66
12	34653.70
13	178.9231

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.149422E-01	-0.105421E-02	-0.714707E-02
3	0.589253E-01	-0.987875E-02	-0.142563E-01
4	0.129477E+00	-0.339057E-01	-0.213028E-01
5	0.222644E+00	-0.799319E-01	-0.282739E-01
6	0.333172E+00	-0.153870E+00	-0.351633E-01
7	0.454693E+00	-0.260536E+00	-0.419651E-01
8	0.579860E+00	-0.403368E+00	-0.486117E-01
9	0.700496E+00	-0.584032E+00	-0.549785E-01
10	0.808353E+00	-0.802894E+00	-0.613227E-01
11	0.895118E+00	-0.105869E+01	-0.673512E-01
12	0.952352E+00	-0.134670E+01	-0.723963E-01
13	0.972583E+00	-0.165575E+01	-0.747875E-01

OPTIMIZATION #3

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loading:

FX.........................	2000.0000
FY.........................	0.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.84614	4.18580	15.45518
2	2.00000	1.68690	4.18580	14.12203
3	2.00000	1.43532	4.18580	12.01594
4	2.00000	1.19577	4.18580	10.01052
5	2.00000	0.97029	4.18580	8.12291
6	2.00000	0.75740	4.18580	6.34069
7	2.00000	0.56761	4.18580	4.75181
8	2.00000	0.40093	4.18580	3.35641
9	2.00000	0.26110	4.18580	2.18585
10	2.00000	0.20000	4.18580	1.67432
11	2.00000	0.20000	4.18580	1.67432
12	2.00000	0.20000	4.18580	1.67432

E) Objective Function:

Total structure Volume: 81.384300

Node	Stress
1	52001.90
2	51862.35
3	51955.02
4	51942.95
5	51853.82
6	51974.19
7	51925.02
8	51871.48
9	51649.30
10	41477.37
11	21322.36
12	9098.637
13	4994.856

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.144840E-01	-0.944371E-03	-0.677752E-02
3	0.557774E-01	-0.914169E-02	-0.131680E-01
4	0.120917E+00	-0.312207E-01	-0.195029E-01
5	0.206307E+00	-0.732726E-01	-0.257570E-01
6	0.307129E+00	-0.140544E+00	-0.319051E-01
7	0.417585E+00	-0.237250E+00	-0.379512E-01
8	0.531063E+00	-0.366366E+00	-0.438425E-01
9	0.640343E+00	-0.529394E+00	-0.495298E-01
10	0.737912E+00	-0.726163E+00	-0.549162E-01
11	0.813769E+00	-0.948447E+00	-0.570368E-01
12	0.860985E+00	-0.118418E+01	-0.577029E-01
13	0.877143E+00	-0.142556E+01	-0.578414E-01

OPTIMIZATION #4

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loading:

FX..........................	0.0000
FY..........................	0.0000
FM..........................	0.0000
FA..........................	-100.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	2.08018	4.18580	17.41440
2	2.00000	1.93027	4.18580	16.15947
3	2.00000	1.76172	4.18580	14.74842
4	2.00000	1.55158	4.18580	12.98917
5	2.00000	1.31450	4.18580	11.00444
6	2.00000	1.06716	4.18580	8.93384
7	2.00000	0.82127	4.18580	6.87536
8	2.00000	0.59123	4.18580	4.94951
9	2.00000	0.39007	4.18580	3.26550
10	2.00000	0.22180	4.18580	1.85686
11	2.00000	0.20000	4.18580	1.67432
12	2.00000	0.20000	4.18580	1.67432

E) Objective Function:

Total structure Volume: 101.545610

Node	Stress
1	52103.21
2	52039.99
3	52033.59
4	52017.27
5	52015.93
6	51958.49
7	51950.00
8	51920.60
9	51610.81
10	51626.45
11	25188.80
12	5614.041
13	1096.744

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	0	0	0

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.000000E+00
2	0.145714E-01	-0.111348E-02	-0.691742E-02
3	0.573794E-01	-0.978175E-02	-0.138459E-01
4	0.125999E+00	-0.332242E-01	-0.206572E-01
5	0.216392E+00	-0.779492E-01	-0.273459E-01
6	0.323304E+00	-0.149537E+00	-0.339032E-01
7	0.440430E+00	-0.252410E+00	-0.403032E-01
8	0.560627E+00	-0.389635E+00	-0.465215E-01
9	0.676171E+00	-0.562738E+00	-0.525061E-01
10	0.778995E+00	-0.771440E+00	-0.581423E-01
11	0.861090E+00	-0.101350E+01	-0.633131E-01
12	0.913642E+00	-0.127777E+01	-0.650579E-01
13	0.931478E+00	-0.154990E+01	-0.651839E-01

OPTIMIZATION #5

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points... 13
No of Degrees of Freedom.... 39
Length per Element.......... 4.1858
Number of Iterations........ 1

C) Structure Loading:

FX......................... 0.0000
FY......................... -8000.0000
FM......................... 0.0000
FA......................... 0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	0.51296	4.18580	4.29430
2	2.00000	0.90694	4.18580	7.59253
3	2.00000	1.11658	4.18580	9.34755
4	2.00C00	1.47904	4.18580	12.38193
5	2.00000	1.19599	4.18580	10.01239
6	2.00000	1.12703	4.18580	9.43503
7	2.00000	1.53118	4.18580	12.81847
8	2.00000	0.57650	4.18580	4.82624
9	2.00000	0.59349	4.18580	4.96845
10	2.00000	1.34594	4.18580	11.26764
11	2.00000	2.20595	4.18580	18.46734
12	2.00000	3.12364	4.18580	26.14985

E) Objective Function:

Total structure Volume: 131.561722

Node	Stress
1	8078.639
2	52000.93
3	50895.98
4	52005.46
5	52048.33
6	52010.10
7	42175.72
8	49131.36
9	4933.140
10	52013.68
11	52043.57
12	52041.75
13	52027.50

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	0
13	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.208648E-01
2	-0.823069E-01	0.426506E-02	0.173341E-01
3	-0.143411E+00	0.157328E-01	0.118840E-01
4	-0.178618E+00	0.270851E-01	0.554722E-02
5	-0.189389E+00	0.319111E-01	0.244944E-04
6	-0.177714E+00	0.234612E-01	-0.683719E-02
7	-0.146174E+00	-0.494672E-02	-0.132138E-01
8	-0.105000E+00	-0.525035E-01	-0.165616E-01
9	-0.619691E-01	-0.118718E+00	-0.201392E-01
10	-0.273824E-01	-0.190906E+00	-0.168476E-01
11	-0.803875E-02	-0.249018E+00	-0.117570E-01
12	-0.764124E-03	-0.286585E+00	-0.606703E-02
13	0.000000E+00	-0.300012E+00	0.000000E+00

OPTIMIZATION #6

OPTIMIZATION SOLUTION

A) Problem Parameters:

Arch Angle :	90.000	Youngs Modulus:	30000000.0
Arch Radius:	32.000	Yield Strength:	52000.0
Arch Height:	2.000	No of Elements:	12

B) Derived Constants:

No of System Nodal Points...	13
No of Degrees of Freedom....	39
Length per Element..........	4.1858
Number of Iterations........	1

C) Structure Loading:

FX.........................	0.0000
FY.........................	-8000.0000
FM.........................	0.0000
FA.........................	0.0000

D) Elemental Dimensions and Stress Distribution:

Element	Height	Base	Length	Volume
1	2.00000	1.72707	4.18580	14.45838
2	2.00000	0.94577	4.18580	7.91762
3	2.00000	0.27194	4.18580	2.27661
4	2.00000	0.65040	4.18580	5.44488
5	2.00000	1.09075	4.18580	9.13130
6	2.00000	0.78986	4.18580	6.61236
7	2.00000	1.06223	4.18580	8.89255
8	2.00000	0.75388	4.18580	6.31116
9	2.00000	0.25849	4.18580	2.16400
10	2.00000	0.93275	4.18580	7.80861
11	2.00000	1.75263	4.18580	14.67236
12	2.00000	2.65143	4.18580	22.19675

E) Objective Function:

Total structure Volume: 107.886574

Node	Stress
1	51991.36
2	49986.21
3	46231.32
4	51991.33
5	49404.85
6	51991.04
7	51841.79
8	42080.21
9	51997.34
10	51279.07
11	52434.15
12	51908.94
13	51935.26

F) Boundary Conditions:

Node	X-Displ	Y-Displ	Slope
1	1	1	1
13	1	0	1

G) Solution Vector:

Node	X-Displ	Y-Displ	Slope

1	0.000000E+00	0.000000E+00	0.000000E+00
2	-0.121854E-01	0.457452E-03	0.527930E-02
3	-0.442765E-01	0.615066E-02	0.947556E-02
4	-0.861533E-01	0.177060E-01	0.907372E-02
5	-0.112986E+00	0.297132E-01	0.435047E-02
6	-0.121058E+00	0.342979E-01	-0.109393E-03
7	-0.110454E+00	0.237526E-01	-0.701848E-02
8	-0.850803E-01	-0.622361E-02	-0.115516E-01
9	-0.537900E-01	-0.547404E-01	-0.155000E-01
10	-0.252856E-01	-0.118485E+00	-0.155501E-01
11	-0.744947E-02	-0.173190E+00	-0.110976E-01
12	-0.654979E-03	-0.209063E+00	-0.580765E-02
13	0.000000E+00	-0.222069E+00	0.000000E+00

APPENDIX E

ARCH_OPT.FOR SOURCE CODE

```
      PROGRAM ARCH_OPTIMIZATION
***********************************************************************
*                                                                     *
*                  ARCH OPTIMIZATION ANALYSIS CODE                    *
*                                                                     *
***********************************************************************
*
*  ALPHA....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO X-AXIS)
*  ANGLE....TOTAL ANGLE OF ARCH (IN DEGREES)
*  BASE.....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS
*  BASEL....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS LOWER
*           SIDE CONSTRAINT
*  BASEU....DOT ARRAY CONTAINING THE ELEMENTAL BASE DIMENSIONS UPPER
*           SIDE CONSTRAINT
*  BETA ....TRANSFORMATION ANGLE OF ELEMENT (ANGLE TO Y-AXIS)
*  B_1......BOUNDARY TERMS APPLIED AT END "1"
*  B_2......BOUNDARY TERMS APPLIED AT END "2"
*  C1,..,C5...CONSTANTS RELATED TO ELEMENT STIFFNESS COEFFICIENTS
*  CLAN.....CONCENTRATED LOAD APPLICATION NODE (THE NODE FX,FY,FM ARE
*           APPLIED
*  COUNT....COUNTS THE NUMBER OF ITERATIONS COMPLETED
*  DOF......DEGREE OF FREEDOMS (UNKNOWN DISPLACEMENTS & SLOPES)
*  DSN......DESIGN VARIABLE FOR EACH ELEMENT
*  DV1BG....DESIGN VARIABLE #1 (BASE DIMENSION) INITIAL ESTIMATE
*  DV1LO....DESIGN VARIABLE #1 (BASE DIMENSION) LOWER SIDE CONSTRAINT
*  DV1UP....DESIGN VARIABLE #1 (BASE DIMENSION) UPPER SIDE CONSTRAINT
*  EK.......6X6 ELEMENT STIFFNESS MATRIX IN LOCAL X,Y COORDINATES
*  EKPR.....6X6 ELEMENT STIFFNESS MATRIX IN ELEMENT LOCAL COORDINATES
*  ELEN.....LENGTH OF ELEMENT
*  F........FORCE VECTOR OF SYSTEM
*  FA.......CONSTANT DISTRIBUTED LOAD IN X DIRECTION FROM END TO END
*  FM.......CONCENTRATED MOMENT AT FREE END
*  FX.......CONCENTRATED LOAD IN X DIRECTION AT FREE END
*  FY.......CONCENTRATED LOAD IN Y DIRECTION AT FREE END
*  G........THE ARRAY OF CONSTRAINT FUNCTIONS
*  GAMMA....6X6 ELEMENT TRANSFORMATION MATRIX
*  GK.......(NDOF)X(NDOF) GLOBAL STIFFNESS MATRIX
*  H........DEPTH OF ARCH SECTION
*  INDSN....INITIAL (UNIFORM) DESIGN DIMENSION
*  INFO.....DOT PARAMETER USED TO SIGNAL THAT THE OPT IS COMPLETE
*  IPRINT...DOT PARAMETER USED SELECT THE DATA OUTPUT FORMAT
*  IPRM.....DOT SELECTABLE INTEGER PARAMETERS
*  ITERATE..THE NUMBER OF TIMES DOT IS TO BE RELOADED WITH THE
*           PRECEEDING DATA
*  IWK......DOT INTERNAL WORK SPACE ARRAY
*  METHOD...DOT PARAMETER USED TO DEFINE THE OPTIMIZATION METHOD
*  MINMAX...DOT PARAMETER USED TO MINIMIZE/MAXIMIZE THE PROBLEM
*  NCON.....NUMBER OF DESIGN CONSTRAINTS
*  NDOF.....NUMBER OF DEGREES OF FREEDOM
*  NEL......NUMBER OF ELEMENTS
*  NRIWK....DOT INTERNAL WORK SPACE ARRAY DIMENSION
*  NRWK.....DOT INTERNAL WORK SPACE ARRAY DIMENSION
```

```
*  NSNP.....NUMBER OF SYSTEM NODAL POINTS
*  OBJ......THE OBJECTIVE FUNCTION OF THE OPTIMIZATION
*  OPTDCS...OPTIMIZATION DECISION TO OPTIMIZE THE PROBLEM OR NOT
*  P1...P4..PARAMETER DIMENSION CORRESPONDING TO THE NEL, NSNP, NCON,
*           AND NDOF, RESPECTIVELY
*  PHI......SUBTENDED ELELENT ANGLE (ALSO, PHIANG IN DEGREES)
*  PRCSN....THE PRECISION DESIRED TO SOLVE THE FEM SYSTEM OF EQUATIONS
*  RADIUS...ARCH RADIUS
*  RPRM.....DOT SELECTABLE REAL PARAMETERS
*  SIGMA_B..THE ELEMENTAL NORMAL STRESS DUE TO BENDING
*  SIGMA_N..THE ELEMENTAL NORMAL STRESS DUE TO AXIAL FORCES
*  SIGMA_T..THE MAXIMUM TOTAL STRESS IN EACH ELEMENT
*  U........THE "DISPLACEMENT" VECTOR OF THE SYSTEM OF LINEAR EQUATIONS
*  WK.......DOT INTERNAL WORK AREA
*  X........GLOBAL HORIZONTAL COORDINATE
*  Y........GLOBAL VERTICAL COORDINATE
*  YIELD....YIELD STRENGTH OF THE ARCH MATERIAL
*  YOUNG....YOUNG'S MODULUS OF THE ARCH MATERIAL
**********************************************************************
C        ....declare the variables...................................
         INCLUDE 'ARCH_COM.FOR'
C
C        ....read the input parameters...............................
         OPEN(8, FILE='ARCH_IN.DAT', STATUS='OLD')
         READ(8,*) ANGLE,RADIUS,YOUNG,YIELD,NEL,METHOD,IPRINT,DV1BG,
     &             DV1LO,DV1UP,H,CLAN,FX,FY,FM,FA,OPTDCS,ITERATE,PRCSN,
     &             BX1,BY1,BM1,BX2,BY2,BM2,LABEL
C
C        ....define constants........................................
         NCON = 3*NEL
         NSNP = NEL + 1
         NDOF = 3*NSNP
C
C        ....determine the system nodal coord and element orientation..
         CALL GEOMETRY (NEL,NSNP,ANGLE,RADIUS,X,Y,ALPHA,BETA,ELEN)
C
C        ....define the size of the work arrays for DOT...............
         NRWK = 38800
         NRIWK =  400
C
C        ....optimize the problem....................................
         CALL OPTIMIZATION_TOOL
C
C        ....compile and format the output...........................
         CALL ARCH_OUTPUT
C
         END
***********************************************************************
*                                                                     *
         SUBROUTINE GEOMETRY (NEL,NSNP,ANGLE,RADIUS,X,Y,ALPHA,BETA,ELEN)
C        ==============================================================
C        | This routine is used by main ARCH_OPTIMIZATION to generate   |
C        | the x-, y-coordinates of each system node, to determine      |
C        | the orientation of each element, and to calculate the        |
C        | length of each element.                                      |
C        ==============================================================
C        ....declare the variables...................................
         INTEGER NEL,NSNP,P1,P2
         PARAMETER (P1=32,P2=33)
         REAL    ANGLE,RADIUS,ELEN,X(P2),Y(P2),ALPHA(P1) BETA(P1),
     &           PI,PHI,ANG,YNUM,XDEN
```

```
      PARAMETER(PI = 3.141593)
C
C     ....determine the geometric constants........................
      PHI = (ANGLE/NEL)*(PI/180.0)
C
      X(1) = 0.0
      Y(1) = 0.0
C
      DO 100 i=2, NSNP
         ANG = (i-1.0)*PHI
        X(i) = RADIUS * (1.0 - COS(ANG))
        Y(i) = RADIUS * SIN(ANG)
        YNUM = (Y(i) - Y(i-1))
        XDEN = (X(i) - X(i-1))
        ALPHA(i-1) = ATAN2(YNUM,XDEN)
         BETA(i-1) = (PI/2.0) - ALPHA(i-1)
  100 CONTINUE
C
C     ....determine the length of each element.....................
      ELEN = SQRT(X(2)**2.0 + Y(2)**2.0)
C
      RETURN
      END
*******************************************************************
*                                                                 *
      SUBROUTINE OPTIMIZATION_TOOL
C     ===========================================================
C     This subroutine directs the program flow optimization decision
C     i.e., optimize the problem or not.  It also serves to set up &
C     execute the DOT optimization software.
C     ===========================================================
C     ....declare the variables....................................
      INCLUDE 'ARCH_COM.FOR'
      INTEGER i
C
C     ....zero out the RPRM and IPRM arrays........................
      DO 100 i=1,20
         RPRM(i) = 0.0
         IPRM(i) = 0
  100 CONTINUE
C
C     ....initialize COUNT.........................................
      COUNT = 0
C
C     ....refine the constraint tolerence..........................
      RPRM(2) = 0.0001
      RPRM(3) = 0.0001
C
C     ....turn off DOT's auto scaling..............................
      IPRM(2) = -1
C
C     ....increase DOT's default number of iterations..............
      IPRM(3) = 1000
      IPRM(8) = 1000
C
C     ....increase DOT's number of consecutive convergence criteria.
      IPRM(4) = 3
      IPRM(9) = 3
C
C     ....define MINMAX=-1 to minimize the objective function......
      MINMAX = -1
```

```
C
C        ....initialize the design variable limits and best guess......
         DO 200 i=1,NEL
             BASE(i) = DV1BG
            BASEL(i) = DV1LO
            BASEU(i) = DV1UP
  200    CONTINUE
C
C        ....make optimization decision................................
         IF (OPTDCS .NE. 1) THEN
            CALL EVAL
            RETURN
         ENDIF
C
C        ....ready to optimize.........................................
         INFO = 0
C
  300    CALL DOT (INFO,METHOD,IPRINT,NEL,NCON,BASE,BASEL,BASEU,OBJ,
     &                             MINMAX,G,RPRM,IPRM,WK,NRWK,IWK,NRIWK)
C
C        ....evaluate the objective function and constraints...........
         IF (INFO .GT. 0) THEN
            CALL EVAL
            GOTO 300
         ENDIF
C
C        ....refine the solution vector by reoptimizing................
         IF (COUNT .LT. ITERATE) THEN
            INFO = 0
            COUNT = COUNT+1
            GOTO 300
         ENDIF
C
         RETURN
         END
***********************************************************************
*                                                                     *
         SUBROUTINE EVAL
C        ==============================================================
C        This subroutine is used to evaluate the Objective function,
C        constraint functions, and side constraints of the optimization
C        problem.
C        ==============================================================
C        ....declare the variables.....................................
         INCLUDE 'ARCH_COM.FOR'
         INTEGER i,j
C
C        ....calculate the objective function..........................
         OBJ = 0.0
C
         DO 100 i=1,NEL
            OBJ = OBJ + BASE(i)*H*ELEN
  100    CONTINUE
C
C        ....initialize the design constraint vector...................
         DO 200 i=1,NCON
            G(i) = 0.0
  200    CONTINUE
C
C        ....determine the design constraints..........................
         CALL ARCH_STRESS
```

```
C
        DO 210 i=1,NEL
           IF (SIGMA_T(i) .GE. SIGMA_T(i+1)) THEN
              SIGMA = SIGMA_T(i)
           ELSE
              SIGMA = SIGMA_T(i+1)
           ENDIF
C
           G(i) = (SIGMA/YIELD) - 1.0
  210   CONTINUE
C
        DO 220 i=1,NEL
              j=i+NEL
           G(j) = (BASE(i)/(3.0*H)) - 1.0
  220   CONTINUE
C
        DO 230 i=1,NEL
              j=i+(2*NEL)
           G(j) = H/(10.0*BASE(i)) - 1.0
  230   CONTINUE
C
        RETURN
        END
***********************************************************************
*                                                                     *
        SUBROUTINE ARCH_STRESS
C       ==============================================================
C       This subroutine is used to perform the Finite Element analysis
C       of the stresses developed in an arch or beam for a given load-
C       ing.
C       ==============================================================
C       ....declare the variables.................................
        INCLUDE 'ARCH_COM.FOR'
        INTEGER IPVT(99)
        REAL    GK(P4,P4),F(P4)
        REAL*8  BK(P4,P4),BF(P4),BU(P4),FAC(9801),WORK(99)

C
C       ....form the element and system matrices.....................
        CALL FORM (NEL,NDOF,ALPHA,BETA,H,ELEN,YOUNG,BASE,GK)
C
C       ....form the Force vector, F..................................
        CALL FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C
C       ....set the boundary conditiona and loads....................
        CALL BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,BY2,BM2)
C
C       ....solve the system of equations............................
        IF (PRCSN .EQ. 2) THEN
C           ....change GK and F arrays to double precision........
            CALL UPSCALE (NDOF,GK,F,BK,BF)
C           ....solve the system of equations....................
            CALL DL2ARG (NDOF,BK,P4,BF,1,BU,FAC,IPVT,WORK)
C           ....change BU array to single presicion..............
            CALL DOWNSCALE (NDOF,BU,U)
        ELSE
C           ....solve the system of equations....................
            CALL L2ARG (NDOF,GK,P4,F,1,U,FAC,IPVT,WORK)
        ENDIF
C
C       ....determine the stress distribution.......................
```

```
      CALL STRESS(X,Y,ALPHA,BETA,U,NEL,ELEN,YOUNG,H,SIGMA_T)
C
      RETURN
      END
**********************************************************************
*                                                                    *
      SUBROUTINE FORM (NEL,NDOF,ALPHA,BETA,H,ELEN,YOUNG,BASE,GK)
C     ================================================================
C     This subroutine is used to construct the global stiffness mat-
C     rix for the arch problem.
C     ================================================================
C     ....declare the variables........................................
      INTEGER NEL,NDOF,NPOW,IEL,I,J,K,II,JJ,KK,III,JJJ,P1,P2,P4
C
      PARAMETER(P1=32,P2=33,P4=99)
C
      REAL    ELEN,H,BASE(P1),ALPHA(P1),BETA(P1),YOUNG,
     &        C1,C2,C3,C4,C5,CA,CB,EKPR(6,6),GAM(6,6),EK(P1,6,6),
     &        GAMMA(P1,6,6),GK(P4,P4),EKGA(6,6),GAEKGA(6,6),
     &        ALPHAI,BETAI
C
C     ....define the constants Cx......................................
      NPOW = 1
      C1 = YOUNG*H/ELEN
      C2 = (H/ELEN)**2.0
      C3 = (H**2.0)/(2.0*ELEN)
      C4 = (H**2.0)/3.0
      C5 = C4/2.0
C
C     ....initialize the EKPR and GAM arrays...........................
      DO 100 II = 1,6
         DO 90 JJ= 1,6
            EKPR(II,JJ) = 0.0
            GAM(II,JJ) = 0.0
   90    CONTINUE
  100 CONTINUE
C
C     ....initialize the EK and GAMMA arrays...........................
      DO 130 IEL = 1,NEL
         DO 120 I = 1,6
            DO 110 J = 1,6
                  EK(IEL,I,J) = 0.0
               GAMMA(IEL,I,J) = 0.0
  110       CONTINUE
  120    CONTINUE
  130 CONTINUE
C
C     ....determine the EKPR matrix....................................
      EKPR(1,1) =  C1
      EKPR(1,4) = -C1
      EKPR(2,2) =  C1*C2
      EKPR(2,3) =  C1*C3
      EKPR(2,5) = -C1*C2
      EKPR(2,6) =  C1*C3
      EKPR(3,2) =  C1*C3
      EKPR(3,3) =  C1*C4
      EKPR(3,5) = -C1*C3
      EKPR(3,6) =  C1*C5
      EKPR(4,1) = -C1
      EKPR(4,4) =  C1
      EKPR(5,2) = -C1*C2
```

```
      EKPR(5,3) = -C1*C3
      EKPR(5,5) =  C1*C2
      EKPR(5,6) = -C1*C3
      EKPR(6,2) =  C1*C3
      EKPR(6,3) =  C1*C5
      EKPR(6,5) = -C1*C3
      EKPR(6,6) =  C1*C4
C
C     ....initialize the GK array.................................
      DO 150 I = 1, NDOF
         DO 140 J = 1, NDOF
            GK(I,J) = 0.0
 140     CONTINUE
 150  CONTINUE
C
C     ....determine the GAMMA matrix..............................
      DO 170 IEL = 1,NEL
         ALPHAI = ALPHA(IEL)
         BETAI = BETA(IEL)
         CA = COS(ALPHAI)
         CB = COS(BETAI)
         GAMMA(IEL,1,1) =  CA
         GAMMA(IEL,1,2) =  CB
         GAMMA(IEL,2,1) = -CB
         GAMMA(IEL,2,2) =  CA
         GAMMA(IEL,3,3) =  1.0
         GAMMA(IEL,4,4) =  CA
         GAMMA(IEL,4,5) =  CB
         GAMMA(IEL,5,4) = -CB
         GAMMA(IEL,5,5) =  CA
         GAMMA(IEL,6,6) =  1.0
 170  CONTINUE
C
C     ....initialize the EKGA and GAEKGA arrays...................
      DO 270 IEL = 1, NEL
         DO 190 III = 1,6
            DO 180 JJJ = 1,6
                 EKGA(III,JJJ) = 0.0
               GAEKGA(III,JJJ) = 0.0
 180        CONTINUE
 190     CONTINUE
C
C     ....determine the EKGA array................................
         DO 220 I = 1,6
            DO 215 J = 1,6
               DO 210 K = 1,6
                  EKGA(I,J) = EKGA(I,J) + EKPR(I,K)*GAMMA(IEL,K,J)
 210           CONTINUE
 215        CONTINUE
 220     CONTINUE
C
C     ....determine the GAEKGA array..............................
         DO 240 I = 1,6
            DO 235 J = 1,6
               DO 230 K = 1,6
                  GAEKGA(I,J) = GAEKGA(I,J)
     &                          +GAMMA(IEL,K,I)*EKGA(K,J)
 230           CONTINUE
 235        CONTINUE
 240     CONTINUE
C
```

```
C        ....copy the GAEKGA array into the EK array..................
            DO 260 I = 1.6
               DO 250 J = 1,6
                  EK(IEL,I,J) = GAEKGA(I,J)
  250          CONTINUE
  260       CONTINUE
  270    CONTINUE
C
C        ....construct the GK matrix...................................
         DO 300 IEL = 1, NEL
            II = 3*(IEL-1)
            DO 290 J = 1, 6
               JJ = II + J
                  DO 280 K = 1, 6
                     KK = II + K
                     GK(JJ,KK) = GK(JJ,KK)
     &                          +EK(IEL,J,K)*(BASE(IEL)**NPOW)
  280             CONTINUE
  290       CONTINUE
  300    CONTINUE
C
         RETURN
         END
*********************************************************************
*                                                                   *
         SUBROUTINE FORCE_VECTOR (NEL,NDOF,ELEN,ALPHA,BETA,FA,F)
C        ============================================================
C        This subroutine is used to construct the force vector for the
C        FEM problem specified.
C        ============================================================
C        ....declare the variables.....................................
         INTEGER NEL,NDOF,i,I1,I2,I3,P1,P4
C
         PARAMETER(P1=32,P4=99)
C
         REAL    ELEN,ALPHA(P1),BETA(P1),FA,F(P4)
C
C        ....form the F-vector.........................................
         F(1) = (ELEN/2.0) * (-COS(BETA(1)))
         F(2) = (ELEN/2.0) * (COS(ALPHA(1)))
         F(3) = (ELEN**2.0)/12.0
C
         DO 100 i=2,NEL
            I1 = (i-1)*3 + 1
            I2 = (i-1)*3 + 2
            I3 = (i-1)*3 + 3
C
            F(I1) = (ELEN/2.0)*(-COS(BETA(NEL)))
     &               +(ELEN/2.0)*(-COS(BETA(NEL-1)))
            F(I2) = (ELEN/2.0)*(COS(ALPHA(NEL)))
     &               +(ELEN/2.0)*(COS(ALPHA(NEL-1)))
            F(I3) = 0.0
  100    CONTINUE
C
         F(NDOF-2) = (ELEN/2.0)*(-COS(BETA(NEL)))
         F(NDOF-1) = (ELEN/2.0)*(COS(ALPHA(NEL)))
         F(NDOF)   = -(ELEN**2.0)/12.0
C
C        ....scale the F-vector by FA..................................
         DO 200 i=1,NDOF
            F(i) = FA*F(i)
```

```
  200   CONTINUE
C
        RETURN
        END
************************************************************************
*                                                                      *
        SUBROUTINE BNDARY (NDOF,GK,CLAN,FX,FY,FM,F,BX1,BY1,BM1,BX2,
     &                                                          BY2,BM2)
C       ==============================================================
C       This subroutine is used to impose the boundary conditions upon
C       the global stiffness matrix and force vector.
C       ==============================================================
        INTEGER NDOF,BX1,BY1,BM1,BX2,BY2,BM2,CLAN,i,N,I1,I2,I3,P4
        PARAMETER(P4=99)
        REAL    GK(P4,P4),FX,FY,FM,F(P4)
C
C       ....invoke the essential boundary conditions..................
        IF (BX1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,1,F)
        ENDIF
C
        IF (BY1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,2,F)
        ENDIF
C
        IF (BM1 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,3,F)
        ENDIF
C
        IF (BX2 .EQ. 1) THEN
           N=NDOF-2
           CALL IMPOSE_BC (NDOF,GK,N,F)
        ENDIF
C
        IF (BY2 .EQ. 1) THEN
           N=NDOF-1
           CALL IMPOSE_BC (NDOF,GK,N,F)
        ENDIF
C
        IF (BM2 .EQ. 1) THEN
           CALL IMPOSE_BC (NDOF,GK,NDOF,F)
        ENDIF
C
C       ....add the concentrated load to the force vector...........
        I1=(CLAN-1)*3+1
        I2=(CLAN-1)*3+2
        I3=(CLAN-1)*3+3
C
        F(I1)=F(I1)+FX
        F(I2)=F(I2)+FY
        F(I3)=F(I3)+FM
C
        RETURN
        END
************************************************************************
*                                                                      *
        SUBROUTINE IMPOSE_BC (NDOF,GK,N,F)
C       ==============================================================
C       This subroutine is used to do the redundant leg work of impos-
C       ing the boundary conditions.
C       ==============================================================
```

```
C       ....declare the variables.....................................
        INTEGER NDOF,N,i,P4
        PARAMETER(P4=99)
        REAL    GK(P4,P4),F(P4)
C
C       ....impose the boundary condition on the GK and F arrays......
        DO 100 i=1,NDOF
           GK(N,i) = 0.0
  100   CONTINUE
           GK(N,N) = 1.0
              F(N) = 0.0
C
        RETURN
        END
**********************************************************************
*                                                                    *
        SUBROUTINE UPSCALE(NDOF,GK,F,BK,BF)
C       ==============================================================
C       This subroutine is used to change the stiffness matrix & force
C       vector from single precision to double precision in order to
C       solve the linear system of equations in double precision.
C       ==============================================================
C       ....declare the variables.....................................
        INTEGER NDOF,i,j,P4
        PARAMETER (P4=99)
        REAL    GK(P4,P4),F(P4)
        REAL*8  BK(P4,P4),BF(P4)
C
C       ....generate the doubleprecision compliments of GK and F......
        DO 110 i=1,NDOF
           DO 100 j=1,NDOF
              BK(i,j) = GK(i,j)
  100      CONTINUE
              BF(i) = F(i)
  110   CONTINUE
C
        RETURN
        END
**********************************************************************
*                                                                    *
        SUBROUTINE DOWNSCALE(NDOF,BU,U)
C       ==============================================================
C       This subroutine is used to do down scale the double precision
C       solution of the linear system of equations back to single pre-
C       cision. DOT could have problems with double precision numbers!
C       ==============================================================
C       ....declare the variables.....................................
        INTEGER NDOF,i,P4
        PARAMETER (P4=99)
        REAL    U(P4)
        REAL*8  BU(P4)
C
C       ....generate the doubleprecision compliments of GK and F......
        DO 100 i=1,NDOF
           U(i) = BU(i)
  100   CONTINUE
C
        RETURN
        END
**********************************************************************
*                                                                    *
```

```
      SUBROUTINE STRESS(X,Y,ALPHA,BETA,U,NEL,ELEN,YOUNG,H,SIGMA_T)
C     ==========================================================
C     This subroutine computes the stress at each nodal point.
C     ==========================================================
C     ....declarations..........................................
      INTEGER NEL,NSNP,NDOF,SWITCH,i,I1,I2,I3,I4,I5,I6,I7,I8,I9,
     &        P1,P2,P4
      PARAMETER(P1=32,P2=33,P4=99)
      REAL    ELEN,H,X(P2),Y(P2),ALPHA(P1),BETA(P1),YOUNG,
     &        K1,K2,CA1,CA2,CB1,CB2,v,v1,v2,
C    &        elxdisp(P2),elydisp(P2),ELEN_f(P2),DISPLEN(P2),
     &        U(P4),SIGMA_N(P4),SIGMA_B(P4),SIGMA_T(P4)
C
C     ....determine the constants...............................
      K1=6.0/(ELEN**2.0)
      K2=2.0/(ELEN)
      NSNP = (NEL + 1)
      NDOF = NSNP*3
C
C     ....determine the bending stresses........................
      DO 100 i=2,NEL
         I1=(i-2)*3+1
         I2=(i-2)*3+2
         I3=(i-2)*3+3
         I4=(i-1)*3+1
         I5=(i-1)*3+2
         I6=(i-1)*3+3
         I7=(i-0)*3+1
         I8=(i-0)*3+2
         I9=(i-0)*3+3
C
         CB1= COS(BETA(i-1))
         CA1= COS(ALPHA(i-1))
         CB2= COS(BETA(i))
         CA2= COS(ALPHA(i))
C
         v2 = K1*(U(I4)-U(I1))*CB1
     &        +K1*(U(I2)-U(I5))*CA1
     &        +K2*(U(I3)+2.0*U(I6))
C
         v1 = K1*(U(I4)-U(I7))*CB2
     &        +K1*(U(I8)-U(I5))*CA2
     &        -K2*(U(I6)*2.0+U(I9))
C
         IF (ABS(v1) .GE. ABS(v2)) THEN
            v=v1
         ELSE
            v=v2
         ENDIF
C
         SIGMA_B(i) = YOUNG*(H/2.0)*v
C
  100 CONTINUE
C
         v = K1*(U(1)-U(4))*COS(BETA(1))
     &      +K1*(U(5)-U(2))*COS(ALPHA(1))
     &      -K2*(U(3)*2.0+U(6))
C
         SIGMA_B(1) = YOUNG*(H/2.0)*v
C
         v = K1*(U(NDOF-2)-U(NDOF-5))*COS(BETA(NEL))
```

```
     &        +K1*(U(NDOF-4)-U(NDOF-1))*COS(ALPHA(NEL))
     &        +K2*(U(NDOF-3)+2.0*U(NDOF))
C
            SIGMA_B(NEL+1) = YOUNG*(H/2.0)*v
C
C     ....determine the normal stresses............................
      SWITCH = 1
C
      IF (SWITCH .EQ. 1) THEN
         DO 300 i=2,NEL
            I1 = (NEL-2)*3+1
            I2 = (NEL-2)*3+2
            I3 = (NEL-2)*3+3
            I4 = (NEL-1)*3+1
            I5 = (NEL-1)*3+2
            I6 = (NEL-1)*3+3
            I7 = (NEL-0)*3+1
            I8 = (NEL-0)*3+2
            I9 = (NEL-0)*3+3
C
            CA1= COS(ALPHA(NEL-1))
            CB1= COS(BETA(NEL-1))
            CA2= COS(ALPHA(NEL))
            CB2= COS(BETA(NEL))
C
            v2 = (U(I4)-U(I1))*CA1 + (U(I5)-U(I2))*CB1
            v1 = (U(I7)-U(I4))*CA2 + (U(I8)-U(I5))*CB2
C
            IF (ABS(v1) .GE. ABS(v2)) THEN
               v=v1
            ELSE
               v=v2
            ENDIF
C
            SIGMA_N(i) = (YOUNG/ELEN)*v
  300    CONTINUE
C
         v = (U(4)-U(1))*COS(ALPHA(1)) + (U(5)-U(2))*COS(BETA(1))
         SIGMA_N(1) = (YOUNG/ELEN)*v
C
         v = (U(NDOF-2)-U(NDOF-5))*COS(ALPHA(NEL)) +
     &       (U(NDOF-1)-U(NDOF-4))*COS(BETA(NEL))
         SIGMA_N(NEL+1) = (YOUNG/ELEN)*v
C
      ELSE
         DO 350 i=1,NEL+1
            SIGMA_N(i) = 0.0
  350    CONTINUE
      ENDIF
C
C     ....determine the total stresses at each node.................
      DO 400 i=1,NEL+1
         SIGMA_T(i) = ABS(SIGMA_B(i)) + ABS(SIGMA_N(i))
  400 CONTINUE
C
      RETURN
      END
************************************************************************
*                                                                      *
      SUBROUTINE ARCH_OUTPUT
C     ==================================================================
```

```
C       This subroutine formats the final results and output of the
C       optimization problem and stores it in a file named ARCH_OUT.DAT
C       ==============================================================
C       ....declare variables.........................................
        INCLUDE 'ARCH_COM.FOR'
        REAL     VOL,VOLUME
C
C       ....open output file and write header.........................
        OPEN(9, FILE='ARCH_OUT.DAT', STATUS='UNKNOWN')
C
        WRITE(9,100) LABEL
        WRITE(9,100) ' OPTIMIZATION SOLUTION'
        WRITE(9,105) ' -----------------------------------------------
     &----------------------'
  100   FORMAT(/5X,A)
  105   FORMAT(5X,A)
C
C       ....section "A"...............................................
        WRITE(9,100) ' A) Problem Parameters:'
        WRITE(9,110) ' Arch Angle :', ANGLE,  ' Youngs Modulus:',YOUNG
        WRITE(9,110) ' Arch Radius:', RADIUS, ' Yield Strength:',YIELD
        WRITE(9,115) ' Arch Height:', H,      ' No of Elements:',NEL
  110   FORMAT(8X,A,F12.3,T38,A,F12.1)
  115   FORMAT(8X,A,F12.3,T38,A,I10)
C
C       ....section "B"...............................................
        WRITE(9,100) ' B) Derived Constants:'
        WRITE(9,120) ' No of System Nodal Points...',NSNP
        WRITE(9,120) ' No of Degrees of Freedom....',NDOF
        WRITE(9,125) ' Length per Element..........',ELEN
C       WRITE(9,125) ' Phi Angle per Element.......',PHIANG
        WRITE(9,120) ' Number of Iterations........',ITERATE

  120   FORMAT(8X,A,I6)
  125   FORMAT(8X,A,F12.4)
C
C       ....section "C"...............................................
        WRITE(9,100) ' C) Structure Loading:'
        WRITE(9,125) 'FX.........................',FX
        WRITE(9,125) 'FY.........................',FY
        WRITE(9,125) 'FM.........................',FM
        WRITE(9,125) 'FA.........................',FA
C
C       ....section "D"...............................................
        WRITE(9,100) ' D) Elemental Dimensions and Stress Distribution:'
        WRITE(9,210) 'Element','Height','Base','Length','Volume'
C
  210   FORMAT(8X,A,T19,A,T34,A,T50,A,T62,A)
  220   FORMAT(8X,I4,T17,F10.5,T32,F10.5,T48,F8.5,T60,F8.5)
        VOLUME = 0.0
C
        DO 300 i=1,NEL
           VOL = H*ELEN*BASE(i)
           WRITE(9,220) i,H,BASE(i),ELEN,VOL
           VOLUME = VOLUME + VOL
  300   CONTINUE
C
C       ....section "E"...............................................
        WRITE(9,100) ' E) Objective Function:'
        WRITE(9,310) ' Total structure Volume:',VOLUME
  310   FORMAT(8X,A,F12.6)
```

```
      WRITE(9,330) 'Node','Stress'
      DO 320 i=1,NSNP
         WRITE(9,*) i,SIGMA_T(i)
 320  CONTINUE
 330  FORMAT(8X,A,T19,A)
C
C     ....section "F"...............................................
      WRITE(9,100) ' F) Boundary Conditions:'
      WRITE(9,410) 'Node','X-Displ','Y-Displ','Slope'
      WRITE(9,430) 1,BX1,BY1,BM1
      WRITE(9,430) NEL+1,BX2,BY2,BM2
C
C     ....section "G"...............................................
      WRITE(9,100) ' G) Solution Vector:'
      WRITE(9,410) 'Node','X-Displ','Y-Displ','Slope'
      DO 400 i=1,NSNP
            I1=(i-1)*3+1
            I2=(i-1)*3+2
            I3=(i-1)*3+3
         WRITE(9,420) i,U(I1),U(I2),U(I3)
 400  CONTINUE
 410  FORMAT(T9,A,T17,A,T31,A,T46,A)
 420  FORMAT(7X,I5,3E14.6)
 430  FORMAT(7X,I5,T20,I4,T34,I4,T48,I4)
C
      RETURN
      END
```

```
C       ARCH_COMMON
C
C       ....definitions........................................
C       P1.....The maximum number of elements
C       P2.....The maximum number of global nodal points
C       P3.....The maximum number of design constraints
C       P4.....The maximum number of degrees of freedom
C
C       ....declare the variables..............................
        INTEGER NEL,NCON,NSNP,NDOF,METHOD,MINMAX,INFO,IPRINT,IWK(400),
     &          NRWK,NRIWK,IPRM(20),COUNT,OPTDCS,ITERATE,PRCSN,CLAN,
     &          BX1,BY1,BM1,BX2,BY2,BM2,P1,P2,P3,P4
C
        PARAMETER(P1=32,P2=33,P3=96,P4=99)
C
        REAL    ANGLE,RADIUS,ELEN,H,X(P2),Y(P2),ALPHA(P1),BETA(P1),
     &          YOUNG,YIELD,WK(27000),RPRM(20),OBJ,G(P3),
     &          DV1BG,DV1LO,DV1UP,BASE(P1),BASEL(P1),BASEU(P1),
     &          FA,FX,FY,FM,U(P4),SIGMA_T(P4)
C
C       ....make in common.....................................
        COMMON  NEL,NCON,NSNP,NDOF,METHOD,MINMAX,INFO,IPRINT,IWK,
     &          NRWK,NRIWK,IPRM,COUNT,OPTDCS,ITERATE,PRCSN,CLAN,
     &          BX1,BY1,BM1,BX2,BY2,BM2,
     &          ANGLE,RADIUS,ELEN,H,X,Y,ALPHA,BETA,YOUNG,YIELD,
     &          WK,RPRM,OBJ,G,DV1BG,DV1LO,DV1UP,BASE,BASEL,BASEU,
     &          FA,FX,FY,FM,U,SIGMA_T
C
```

LIST OF REFERENCES

1. Farshad, M., "On Optimal Form of Arches," *Journal of the Franklin Institute*, pp. 187-194, August 1976.

2. Rozvany, G.I.N., Wang, C.M., and Dow, M., "Arch Optimization via Prager-Shield Criteria," *Journal of the Engrg. Mech. Div., ASCE*, 106, pp.1279-1286, December 1980.

3. Lipson, S.L., and Muhammad, I.H., "Optimal Design of Arches Using the Complex Method," *Journal of the Structural Div., ASCE*, ST12, pp. 2509-2525, December 1980.

4. Ang, B.W., Teo, K.L., and Wang, C.M., "Optimal Shape of Arches under Bending and Axial Compression," *Journal of the Engrg. Mech. Div., ASCE*, 114, pp. 898-905, May 1988.

5. *DOT Users Manuel*, Version 2.04, VMA Engineering, 1989.

6. Gere, J.M., and Timoshenko, S.P., *Mechanics of Materials*, 2nd ed., pp. 616,737, PWS-Kent Publishing Co., 1984.

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